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# The role of the shear and normal stress gradients on life estimation of notched Al7050-T7451 under multiaxial loadings

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ABSTRACT

The aim of this work was to investigate the role of the normal and shear stress gradients on the life estimation of notched Al 7050-T7451 components under multiaxial stress states. In this setting, a new concept of an equivalent critical distance, $L_{eq}$, was proposed. To validate the analysis a wide number of experimental fatigue data were generated for smooth and notched specimens under push–pull, torsion and combined in phase axial–torsional loadings. From these data, critical distance versus life curves ($L$–$N_f$) were obtained under torsion and push–pull loading conditions. The Modified Wöhler Curve Method (MWCM) critical plane based multiaxial model was used in conjunction with these curves and with the new $L_{eq}$–$N_f$ relationship to estimate lives. The results showed that the $L$–$N_f$ curves obtained from the fully reversed torsion and push–pull tests had opposite behaviours. The best life estimates for the notched specimens under combined in phase axial–torsional loadings were obtained considering the new $L_{eq}$–$N_f$ relationship curve proposed in this work.

## 1. Introduction

Due to their low density and good mechanical resistance aluminium alloys have been widely used in the aeronautical industry. The Al 7050-T7451 alloy was developed in the 1970s and has been used not only in the fuselage but also to build structural parts of the wings and landing gears of aircrafts. These mechanical components invariably contain some sort of geometrical discontinuity and are submitted to the combined action of loadings, which will produce stress gradients and a complex multiaxial state of stress. Under multiaxial fatigue conditions, there are few data available in the literature for such alloy. For instance, Chen et al. [1] reported experimental data on Al 7050-T7451 under combined loadings, however, these data were produced under a low cycle fatigue regime and no stress gradient was present.

There are several criteria published in the literature capable to estimate fatigue life (or strength) of components subjected to a multiaxial state of stress [2–10]. However, these models were developed to deal with smooth specimens/components under a uniform stress field. The introduction of stress gradient effects caused by notches (or contact mechanics, etc) into such models is still a matter of strong debate among fatigue scientists. In the medium-high cycle fatigue regime (MHCF) the analysis of notched components, i.e. in the presence of high stress gradients, can be conducted by using the Theory of Critical Distances (TCD) associated with a multiaxial fatigue model. The TCD considers that the fatigue strength of a mechanical component under a rapidly varying stress field depends on an average state of elastic stress over a process zone. Tanaka [11] and Taylor [12] initially suggested that the size of such zone could be extracted from the intrinsic crack length, as defined by El Haddad et al. [13]. Further, Taylor et al. [14] proposed that the effective stress, which should control fatigue damage, and hence that needed to be averaged over the process zone, was the amplitude of the maximum principal stress. In the setting of multiaxial fatigue, Susmel and Taylor [15] applied the Modified Wöhler Curve Method (MWCM) [6] in terms of the TCD to estimate the fatigue limit of notched components under combined loadings. Araújo et al. [16,17] also demonstrated that this approach was successful to estimate the fatigue limit of an Al4%Cu aeronautical alloy under fretting conditions within a ± 20% error band. Concerning the size of the process zone, Castro et al. [18] showed that such dimension was not always given by the intrinsic crack length, but its calibration depended on (i) the multiaxial model used as the effective stress in the application of the TCD and (ii) on two fatigue tests under threshold conditions, namely the fatigue limit (obtained from tests on smooth specimens) and the threshold stress intensity factor range (obtained from a cracked sample).

The extension of the TCD to estimate fatigue life in notched components was introduced by Susmel and Taylor [19,20]. In this case, these authors assumed that the critical distance could vary with life and

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**Nomenclature**

| | |
|---|---|
| $A_l$ | coefficient of the function that characterize the decay $\hat{\sigma}_{1a}(l)$ |
| $b_l$ | exponent of the function that characterize the decay $\hat{\sigma}_{1a}(l)$ |
| $A_{\sigma_g}$ | coefficient of the Wöhler curve obtained under push-pull load for the notched specimens |
| $b_{\sigma_g}$ | exponent of the Wöhler curve obtained under push-pull load for the notched specimens |
| $l$ | distance from the root of the notch measured along the bisector plane |
| $l_{trial}$ | hypothetical critical distance |
| $L$ | critical distance |
| $L_{eq}$ | equivalent critical distance |
| $N_o$ | largest number of cycles registered |
| $N_s$ | smallest number of cycles registered |
| $N_{Ref}$ | reference number of cycles to failure of the modified Wöhler curve |
| $N_{f,e}$ | estimated number of cycles to failure |
| $N_f$ | number of cycles to failure |
| $X$ | weight parameter of the equivalent critical distance function |
| $\Delta$ | material plane |
| $\sigma_{1a}$ | amplitude of the maximum principal stress |
| $\hat{\sigma}_{1a}$ | variation of $\sigma_{1a}$ normalized with respect to the amplitude of the normal stress referred to the gross area |
| $\sigma_{ag}$ | amplitude of the normal stress referred to the gross section area |
| $\sigma_{n,max}$ | maximum value of stress component $\sigma_n$ |
| $\tau_{A,Ref}$ | reference shear stress amplitude at $N_A$ cycles to failure, |
| $\tau_a$ | amplitude of shear stress |
| $\tau_{ag}$ | amplitude of the shear stress referred to the gross area |
| $\kappa$ | negative inverse slope of the modified Wöhler curve |
| $\lambda$ | shear and the normal stress amplitude ratio |
| $\rho$ | stress ratio relative to the critical plane |
| $\omega$ | angular velocity |
| $\phi$ | phase angle |

hence they proposed there was a simple linear relationship between them (in logarithmic scale). To obtain such relationship it is usually necessary to generate two S-N curves (under push-pull conditions) for plain (smooth) and for notched specimens. This essentially means that only the gradient of normal stresses is accounted for in the determination of the critical distance/process zone size. Furthermore, it is worthwhile noticing that, to calibrate the size of the critical distance as a function of life in such approach, Susmel and Taylor [19,20] considered only the gradient of the amplitude of maximum principal stress along the bisector line of the notch, however the effective stress used to conduct the average over the process zone is that given by the MWCM [6]. The immediate question which arises here is: why the process zone size for materials in which the crack initiation process is shear dominated would depend only on one set of data extracted under push-pull conditions and vice versa? Therefore, the purpose of this work is to investigate the role of combined shear and normal stress gradients in such a life methodology. In order to do so, the critical distance against life relationship will be raised not only for fully reversed push-pull tests but also for alternated torsion and an equivalent measure for the critical distance will be proposed. To validate the analysis multiaxial fatigue tests will be conducted on notched Al 7050-T7451 specimens.

## 2. Fatigue damage parameter – Modified Wöhler curve method

The use of multiaxial models based on critical plane approaches has become increasingly important due to the good results obtained in predicting the fatigue life. The fundamental hypothesis of the critical plane models is that the orientation of the microcracks can be identified by searching for the *most damaged material plane*. The quantification of the level of fatigue damage introduced into a plane is expressed as a function of a *damage parameter*, which is usually represented by a combination of the normal and the shear stress components acting on such a plane. In this study, the Modified Wöhler Curve Method [6] was chosen to conduct the life analysis for notched specimens subjected to combined axial-torsional loadings in the medium high cycle fatigue regime (MHCF).

The method assumes that the critical plane is the one under the largest amplitude of shear stress, $\tau_a$. The maximum normal stress, $\sigma_{n,max}$, on this most damaged plane also contributes to crack formation. Graphically, the model can be represented in a diagram of $\tau_a$ against the number of cycles to failure, $N_f$. This so called modified Wöhler curve diagram contains different fatigue curves characterized by different values of $\rho$ ratio, defined as:

$$\rho = \frac{\sigma_{n,max}}{\tau_a} \tag{1}$$

Each modified Wöhler curve is defined by its negative inverse slope, $\kappa$, and by a reference shear stress amplitude, $\tau_{A,Ref}$, extrapolated at an appropriate number of cycles to failure, $N_{Ref}$. For a given material, the corresponding modified Wöhler diagram can directly be built provided that the $\kappa$ *vs* $\rho$ and $\tau_{A,Ref}$ *vs* $\rho$ relationships are calibrated by running at least two sets of basic experiments, such as fully reversed push-pull ($\rho = 1$) and fully reversed torsion ($\rho = 0$) as a function of life. After determining the appropriate material constants (see [7] for more details) any intermediate curve can be obtained. From the specific modified Wöhler curve for the ρ ratio that characterizes the local stress history being assessed, the number of cycles to failure can then be estimated as:

$$N_f = N_{Ref}\left[\frac{\tau_{A,Ref}(\rho)}{\tau_a}\right]^{\kappa(\rho)} \tag{2}$$

where $N_{Ref}$ is the reference number of cycles to failure (e.g., $N_{Ref} = 2 \times 10^6$ cycles). $\tau_{A,Ref}(\rho)$ and $\kappa_\tau(\rho)$ can be calculated as:

$$\tau_{A,Ref}(\rho) = [\tau_{A,Ref}(\rho = 1) - \tau_{A,Ref}(\rho = 0)]\rho + \tau_{A,Ref}(\rho = 0) \tag{3}$$

$$\kappa_\tau(\rho) = [\kappa_\tau(\rho = 1) - \kappa_\tau(\rho = 0)]\rho + \kappa_\tau(\rho = 0) \tag{4}$$

## 3. New discussion on the definition of the TCD to estimate fatigue life of notched components under multiaxial loads

### 3.1. Classical approach

The central idea in the TCD [9,11,12,14,15,19,20] is the definition of an effective stress, based on an averaging procedure over a volume surrounding the stress raiser. Fatigue failure is expected to occur if this effective stress exceeds a reference material fatigue strength. Simplified methods can also be formulated by considering averages over an area or a line (Area and Line Methods, respectively) or the stress at a point located at a critical distance, $L$, from the stress raiser (Point Method). The Point Method is used in this work.

At the medium-cycle fatigue regime, $L$, depends on the number of cycles to failure, $N_f$ [19]. A power law relationship can be written as:

$$L(N_f) = AN_f^b \tag{5}$$

where $A$ and $b$ are material constants. The fitting procedures to obtain these constants require two fatigue curves generated by testing plain and sharply notched specimens under fully reversed push-pull