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Analytical solution with validity analysis for an elliptical void and a rigid inclusion under uniform or nonuniform anti-plane loading

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Abstract

An elliptical inclusion (covering both void and rigid inclusions) embedded in an infinite and finite elastic plane subject to uniform and nonuniform (m-th order polynomial) anti-plane loading conditions is analyzed. An analytical solution in terms of the stress field for an infinite plane is developed through the method of analytic function and conformal mapping. Closed-form complex potentials and analytical expressions for Stress Concentration Factors (SCFs) are obtained. The results show that (i.) the SCF value decreases with an increasing loading order, so that the influence of the non-uniformity of the anti-plane loads on the SCF is revealed to be beneficial from the failure point of view; (ii.) decrease in the SCF value for an infinite plane is monotonic, which does not hold true for a finite plane. The results for an infinite plane are confirmed and extended for finite planes by exploiting the well-known heat–stress analogy and the finite element method. It is worth mentioning that the comparison between the analytical solution for an infinite plane and the numerical solution for finite plane is provided, showing that the analytical solution of an infinite plane can be used as an accurate approximation to the case of a finite plane. Moreover, the proposed heat–stress analogy can be exploited to study the crack–inclusion interaction or multiply connected bodies. The computational efficiency of the proposed methodology makes it an attractive analysis tool for anti-plane problems with respect to the full scale three-dimensional analysis.

Keywords: anti-plane elasticity, SCF, SIF, crack, heat–stress analogy, composites, Laplace equation.

1 Introduction

The increasing demand of lightweight high-strength composite materials is undeniable in a wide range of applied sciences such as structural, mechanical, aerospace, nuclear and chemical engineering. On the other hand, voids, cracks, defects and inclusions in structures are known to generate stress concentrations and they are detrimental to the overall strength and durability of structure. For instance, composite materials are susceptible to premature failure during their service time due to the stress concentration around voids or fibers leading to the formation of cracks or micro-cracks. The evolution of these cracks often leads to catastrophic failures [1, 2, 3]. Therefore, the determination of the stress fields near a hole or an inclusion (in an elastic plane) is a key problem for the engineering design of high-strength composite materials [4].

The problem of stress concentration around holes or inclusions under plane and anti-plane loading has been under intense investigation [5, 6, 7, 8, 9, 10] but mostly analytical solutions have been derived primarily for an infinite plane subject to uniform loading conditions. An experimental proof of stress concentrations around circular holes has been provided in [5, 11] and around polygonal inclusions in [12, 13] through photoelastic experiments. In addition, real world experiments on mortar specimens containing cylindrical inclusions have been conducted in [14] showing that the stress concentrations lead to crack openings around the inclusion.

Let us now restrict our attention to the problem of anti-plane elasticity which sometimes is considered to be a sort of a mathematical abstraction, yet the anti-plane problem of an inclusion embedded in an infinite elastic plane is fundamental in fiber-reinforced composites and crack problems [15]. Some simple real life examples of anti-plane