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Phase field modelling of crack propagation, branching and coalescence in rocks

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ABSTRACT

We present a phase field model (PFM) for simulating complex crack patterns including crack propagation, branching and coalescence in rock. The phase field model is implemented in COMSOL and is based on the strain decomposition for the elastic energy, which drives the evolution of the phase field. Then, numerical simulations of notched semi-circular bend (NSCB) tests and Brazil splitting tests are performed. Subsequently, crack propagation and coalescence in rock plates with multiple echelon flaws and twenty parallel flaws are studied. Finally, complex crack patterns are presented for a plate subjected to increasing internal pressure, the (3D) Pertersson beam and a 3D NSCB test. All results are in good agreement with previous experimental and numerical results.

1. Introduction

Fracture-induced failure has gained extensive concern in engineering because of the huge threat to engineering safety [3]. The prediction of fracture in rock is challenging. Rock masses have many pre-existing flaws, such as micro cracks, voids and soft minerals. Many efforts have been made to study crack propagation in rock, see for instance the contributions in Bobet and Einstein [12], Wong et al. [56], Sagong and Bobet [49], Wong and Einstein [55], Park and Bobet [33], Park and Bobet [34], Lee and Jeon [25], and Zhou et al. [59]. However, many studies focus on uniaxial compressive loads since tensile loads or more complicated load cases, which are more difficult to perform in practical tests.

Numerical methods are a good alternative to study fracture problems. They are less expensive than experimental tests and can provide physical insight difficult to gain through 'pure' experimental testing. Computational methods for fracture can be classified in discrete and continuous approaches. Efficient remeshing techniques [6,8,5], multiscale method [16,17,57], strain-softening element [7], the extended finite element method [32,30], the phantom node method [46,19,54], multiscale methods [51,16,17] and specific meshfree methods [37,43,40,46,42,44,1] are classical representatives of the first class. The cracking particles method (CPM) [38,39,45], Peridynamics [41] and dual-horizon peridynamics [47,48] are also discrete crack approaches but they share the simplicity of continuous approaches to fracture as they also do not require any explicit representation of the crack surface and any crack tracking algorithms. Element-erosion [11,24] directly sets the stresses of the elements to zero when the elements fulfill the fracture criterion. However, the element-erosion method cannot simulate crack branching correctly [50]. Gradient models [52], non-local models [36], models based on the screend-poisson equation [4] and also phase field models are typical continuous approaches to fracture.

In this paper, we pursue the phase field model (PFM) [15,28,29,23,13] to model crack propagation, branching and coalescence in rock. The origins of the PFM can be traced back to Bourdin et al. [15], but a thermodynamic consistent framework was first presented by Miehe, Hofacker and Welschinger [28]. Considerable attention has been paid to PFMs due to their ease in implementation and applicability to multi-physics problems. The PFM does not treat the crack as a physical discontinuity but uses a scalar field (the phase field) to smoothly transit the intact material to the broken one. Thus, the sharp crack is represented by a 'damage-like' zone. The shape of the crack is controlled by a length scale parameter and propagation of the crack is obtained through the solution of a differential equation. Thus, the PFM does not require any external criterion for fracture and additional work to track the fracture surface algorithmically [13]. It is

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believed that for this reason, the phase field is therefore has some advantage over other approaches in modeling branching and merging of multiple cracks.

Phase field models have been discretized in the context of the finite element method [9], meshfree methods [2] and isogeometric analysis [13]; the latter two approaches use a fourth-order differential equation for the phase field exploiting the higher continuity of the meshfree and isogeometric approximation. The PFM for brittle cracks has also been implemented in commercial software such as ABAQUS [31,26]. However, the extension of the implementation in ABAQUS to problems with more fields – as hydraulic fracturing – is difficult. Hence, we present an implementation of the phase field model in COMSOL Multiphysics, a software particularly dedicated to multi-field modeling.

This paper is organized as follows. The phase field model for brittle fractures is presented in Section 2. Subsequently, the numerical implementation of the phase field model in COMSOL is described in Section 3. Then, simulations of initiation, propagation, branching, and coalescence of cracks in rock are shown in Section 4 before Section 5 concludes our manuscript.

2. Theory of phase field modeling

2.1. Theory of brittle fracture

Consider an elastic body $\Omega \subset \mathbb{R}^d$ ($d \in \{1,2,3\}$) as shown in Fig. 1, whose external boundary and internal discontinuity boundary are denoted as $\partial\Omega$ and Γ, respectively; $\boldsymbol{x}$ is the position vector and $\boldsymbol{u}(\boldsymbol{x},t) \subset \mathbb{R}^d$ the displacement vector at time t. In Fig. 1, the body Ω satisfies the time-dependent Dirichlet boundary conditions ($u_i(\boldsymbol{x},t) = g_i(\boldsymbol{x},t)$ on $\partial\Omega_{g_i} \in \Omega$), and also the time-dependent Neumann conditions on $\partial\Omega_{h_i} \in \Omega; \boldsymbol{b}(\boldsymbol{x},t) \subset \mathbb{R}^d$ is the body force and $\boldsymbol{f}(\boldsymbol{x},t)$ the traction on boundary $\partial\Omega_{h_i}$.

Given that the stored elastic energy can be transformed into dissipative forms of energy, the classical Griffith's theory [3] for brittle fracture states that the crack starts to propagate when the stored energy is sufficient to overcome the fracture resistance of the material. Therefore, the crack propagation is regarded as a process to minimize a free energy L that consists of the kinetic energy $\Psi_{kin}(\dot{\boldsymbol{u}})$, elastic energy Ψ_ε, fracture energy Ψ_f and external work W_{ext}:

$$L = \Psi_{kin}(\dot{\boldsymbol{u}}) - \underbrace{\int_\Omega \psi_\varepsilon(\boldsymbol{\varepsilon})\mathrm{d}\Omega}_{\Psi_\varepsilon} - \underbrace{\int_\Gamma G_c \mathrm{d}S}_{\Psi_f} + \underbrace{\int_\Omega \boldsymbol{b}\cdot\boldsymbol{u}\mathrm{d}\Omega + \int_{\partial\Omega_{h_i}} \boldsymbol{f}\cdot\boldsymbol{u}\mathrm{d}S}_{W_{ext}} \tag{1}$$

where $\dot{\boldsymbol{u}} = \frac{\partial \boldsymbol{u}}{\partial t}$, ψ_ε is the elastic energy density, and G_c is the critical energy release rate. The linear strain tensor $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\boldsymbol{u})$ is given by

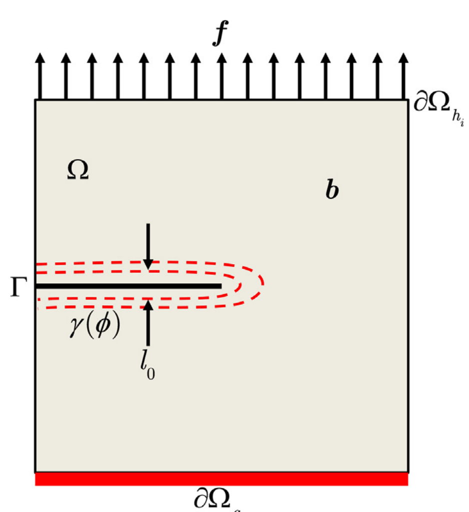

Fig. 1. Phase field approximation of the crack surface.

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^{\mathrm{T}}] \tag{2}$$

The kinetic energy is given by

$$\Psi_{kin}(\dot{\boldsymbol{u}}) = \frac{1}{2}\int_\Omega \rho\dot{\boldsymbol{u}}^2 \mathrm{d}\Omega \tag{3}$$

where ρ indicates the density.

2.2. Phase filed approximation for the fracture energy

The phase field method [28,29,13] uses a scalar field, i.e. the phase field, to smear out the crack surface (see Fig. 1) over the domain Ω. The phase field $\phi(\boldsymbol{x},t) \in [0,1]$ has to satisfy the following conditions:

$$\phi = \begin{cases} 0, & \text{if material is intact} \\ 1, & \text{if material is cracked} \end{cases} \tag{4}$$

A typical one dimensional phase field approximated by the exponential function is given by [28]

$$\phi(x) = e^{-|x|/l_0} \tag{5}$$

l_0 denoting the length scale parameter, which controls the transition region of the phase field and thereby reflects the width of the crack. The distribution of the one dimensional phase field is shown in Fig. 2. The crack region will have a larger width as l_0 increases and the phase field will represent a sharp crack when l_0 tends to zero.

It can be shown that the crack surface density per unit volume of the solid is given by [28]

$$\gamma(\phi,\nabla\phi) = \frac{\phi^2}{2l_0} + \frac{l_0}{2}|\nabla\phi|^2 \tag{6}$$

Thus, the fracture energy is approximated by

$$\int_\Gamma G_c \mathrm{d}S = \int_\Omega G_c\left(\frac{\phi^2}{2l_0} + \frac{l_0}{2}|\nabla\phi|^2\right)\mathrm{d}\Omega \tag{7}$$

The variational approach [14] states that the crack surface energy is transformed from the elastic energy, which drives the evolution of the phase field. To capture cracks only under tension, the elastic energy is decomposed into tensile and compressive parts [29]:

$$\boldsymbol{\varepsilon}_\pm = \sum_{a=1}^{d} \langle\varepsilon_a\rangle_\pm \boldsymbol{n}_a \otimes \boldsymbol{n}_a \tag{8}$$

where $\boldsymbol{\varepsilon}_+$ and $\boldsymbol{\varepsilon}_-$ are the tensile and compressive strain tensors, respectively. In addition, ε_a is the principal strain and $\boldsymbol{n}_a$ is the direction of the principal strain. The operators $\langle\cdot\rangle_\pm$ are defined as: $\langle\cdot\rangle_\pm = (\cdot \pm |\cdot|)/2$. Consequently, the positive and negative elastic energy densities are expressed as

$$\psi_\varepsilon^\pm(\boldsymbol{\varepsilon}) = \frac{\lambda}{2}\langle\mathrm{tr}(\boldsymbol{\varepsilon})\rangle_\pm^2 + \mu\mathrm{tr}(\boldsymbol{\varepsilon}_\pm^2) \tag{9}$$

where $\lambda > 0$ and $\mu > 0$ are the Lamé constants. The Lamé constants are

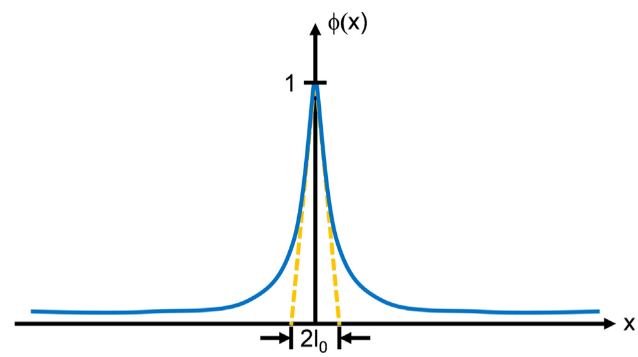

Fig. 2. Distribution of the one dimensional phase field across a crack.