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# Interface crack between magnetoelectroelastic and orthotropic half-spaces under in-plane loading

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ABSTRACT

In this paper, an interface crack between magnetoelectroelastic and orthotropic half-spaces has been studied in detail. By using integral transform techniques the present mixed boundary value problem was reduced to the solution of singular integral equations, which can be further reduced to solving a Riemann-Hilbert problem with closed form solution. The crack-tip singularities of the interface crack have been investigated for possible combinations of the magnetoelectroelastic and orthotropic materials, a criterion based on the coefficient of the Riemann-Hilbert problem is introduced to study the possible singularity behavior of the interface crack, and it is shown that there can be either oscillatory or non-oscillatory singularity for the interface crack depending on the particular combination of the bi-materials. A closed form solution for stresses, electric fields, magnetic fields, electric displacement and magnetic induction in the cracked biomaterials is given, and of particular interests, the analytical expression of the stresses, electric displacements and magnetic inductions along the interface has been obtained.

## 1. Introduction

The widespread use of composite materials in structural applications has encouraged the interests in studying the problem of interfacial cracks between dissimilar isotropic materials, and the nonstandard oscillatory square root singularity of some interface cracks was determined [1–4]. Investigations of an interlaminar crack or delamination between dissimilar, strongly anisotropic composite materials can be found in the works of some researchers [5–10], among others. Due to materials' inherent anisotropy, the interface crack problem involves not only the inherent singular crack-tip fields, but also the significant coupling between in-plane and interlaminar deformations in the composites, i.e., the simultaneous existence of mode I, II, and III fracture [11].

Considering the wide application of piezoelectric materials in smart structures, the problems of interface cracks between dissimilar piezoelectric materials have received considerable attention. Possible oscillatory or non-oscillatory stress singularities for the interfacial cracks in bonded piezoelectric half-spaces have been investigated by Kuo and Barnett [12]. Suo et al. [13] dealt with impermeable interface cracks between dissimilar piezoelectric materials and the solutions showed that the crack tip singularity could be either oscillatory or non-oscillatory. In many practical applications, piezoelectric materials are bonded to non-piezoelectric (conducting or insulating) materials. Parton [14] analyzed interfacial cracks between piezoelectric and conducting isotropic materials. Liu and Hsia [15] studied an interfacial external crack between piezoelectric and orthotropic half-spaces using the method of singular integral equations and Riemann boundary value problems. Ou and Chen [16] and Li and Chen [17] have investigated interfacial cracks between piezoelectric material and non-piezoelectric (insulating or conducting), isotropic, elastic materials by employing the Stroh formalism [18] and assuming that elastic materials have extremely low but non-zero piezoelectric and dielectric constants. It is noted that when a non-piezoelectric elastic material is being treated as a special case of piezoelectric materials with vanishing piezoelectric constants, the solution becomes complicated due to the appearance of repeated eigenvalues [19]. A hybrid complex-variable solution for piezoelectric/isotropic elastic interfacial cracks has been obtained by combining the Stroh's method of piezoelectric materials with Muskhelishvili's method for isotropic elastic materials [20], and a simple explicit condition is given for the absence of the oscillating singularity for interfacial cracks [21].

Composite materials consisting of piezoelectric and piezomagnetic phases have gained considerable interests in recent decades with

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increasing wide application in engineering [22]. Some defects (such as dislocations and cracks) could be induced during the manufacturing processes or during service by the mechanical, electric or magnetic loadings, which can adversely influence the performance of the structures. There has been a growing interest among researchers in solving fracture mechanics problems in magnetoelectroelastic media to study the coupling effect and multi-physical fields on the crack behavior [23–35]. An explicitly analytic solution for an electrically permeable interface crack between two dissimilar magnetoelectroelastic solids has been obtained by Gao et al. [36] using the Stroh formalism, and the results show the singular and oscillatory fields ahead of the crack tips. Feng and Zou [37] examined the dynamic response of an impermeable interface crack between two dissimilar magneto-electro-elastic materials subjected to mechanical, electric and magnetic impacts. The mixed mode I and II interface crack in piezoelectromagneto-elastic anisotropic biomaterials was investigated by Li and Kardomateas [38]. An analyzing method for planar interface cracks of arbitrary shape in three-dimensional, transversely isotropic, magnetoelectroelastic biomaterials has been provided by Zhao et al. [39]. A two-dimensional fracture problem of periodically distributed interface cracks in multilayered piezomagnetic/piezoelectric composites under in-plane loading has been studied by Tian et al. [40], and it is found that a magnetic or electric loading normal to the crack surfaces may lead to the mixture of mode I and mode II type stress singularities at the crack tips. In reality, the interface crack between magnetoelectroelastic and non-magnetoelectroelastic materials has important engineering application background, for example, smart structures or components made of magnetoelectroelastic materials may rest on or attached to orthotropic elastic foundations, and crack may appear on the interface across which material properties change abruptly. Due to the complexity of the interface crack problem, no research work has been reported for this kind of interface crack problem, to the best knowledge of the authors.

In this paper, we develop an exact method for a conducting interface crack between magnetoelectroelastic and orthotropic, elastic materials by using the method of Integral transforms and singular integral equations, and closed-form solutions have been obtained by solving the corresponding Riemann-Hilbert problem. A closed-form solution of the fields in the cracked bi-materials has been obtained, and the analytical solution for the stresses, electric displacements and magnetic induction on the bonded interface is obtained. The Riemann-Hilbert problem is investigated in detail to study the possible singularity behavior of the interface crack, and it shows that the interface crack can have either oscillatory or non-oscillatory singularities depending on the value of the coefficient $(1-q)/(1+q)$, which is a function of the material properties of the bi-materials. The extended Dunders parameter $\beta$ for the interface crack [41] between magnetoelectroelastic and orthotropic materials has been obtained and the relation between $\beta$ and the $q$ -parameter was given. Numerical results of the singularity parameter have been provided for different combinations of the magnetoelectroelastic and orthotropic materials. The distributions of the singular stresses, electric displacements and magnetic induction near the interface crack tip have been displayed graphically.

## 2. Problem statement

We study an interface crack of length $2c$ between magnetoelectroelastic and orthotropic half-spaces, with the poling direction of the magnetoelectroelastic medium perpendicular to the crack plane, as shown in Fig. 1. For convenience, a set of Cartesian coordinate system $(x, y)$ is attached to the crack. Assume that a uniform normal stress, $P_0$, is applied on the crack faces.

Consider a transversely isotropic, linear elastic magnetoelectroelastic half-space and denote the rectangular coordinates of a point by $(x, y)$. In the absence of body forces and electric charge density, the equilibrium equations for plane strain, magnetoelectroelastic material can be expressed as

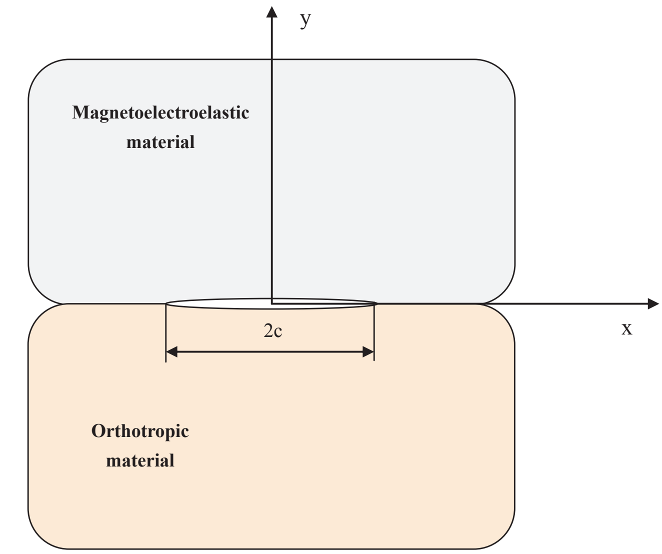


**Fig. 1.** The interface crack between magnetoelectroelastic and orthotropic materials.

$$\begin{aligned}
&C_{11}u_{x,xx} + C_{44}u_{x,yy} + (C_{13} + C_{44})u_{y,xy} + (e_{31} + e_{15})\phi_{,xy} + (h_{31} + h_{15})\varphi_{,xy} = 0\\
&(C_{13} + C_{44})u_{x,xy} + C_{44}u_{y,xx} + C_{33}u_{y,yy} + e_{15}\phi_{,xx} + e_{33}\phi_{,yy} + h_{15}\varphi_{,xx} + h_{33}\varphi_{,yy} = 0\\
&(e_{31} + e_{15})u_{x,xy} + e_{15}u_{y,xx} + e_{33}u_{y,yy} - \lambda_{11}\phi_{,xx} - \lambda_{33}\phi_{,yy} - d_{11}\varphi_{,xx} - d_{33}\varphi_{,yy} = 0\\
&(h_{31} + h_{15})u_{x,xy} + h_{15}u_{y,xx} + h_{33}u_{y,yy} - d_{11}\phi_{,xx} - d_{33}\phi_{,yy} - \mu_{11}\varphi_{,xx} - \mu_{33}\varphi_{,yy} = 0
\end{aligned} \tag{1}$$

where $u_x$, $u_y$ are components of the displacement vector, $\phi$ is the electric potential, $\varphi$ is the magnetic potential, $C_{11}$, $C_{13}$, $C_{33}$, $C_{44}$ are elastic constants, $e_{15}$, $e_{31}$ are piezoelectric constants, $h_{15}$, $h_{31}$ are piezomagnetic constants, $\lambda_{11}$, $\lambda_{33}$ are dielectric permittivities, $d_{11}$, $d_{33}$ are electromagnetic constants, and $\mu_{11}$, $\mu_{33}$ are magnetic permeabilities.

The constitutive equations of the magnetoelectroelastic media under plain strain are given as follows

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \begin{bmatrix}C_{11} & C_{13} & 0\\ C_{13} & C_{33} & 0\\ 0 & 0 & C_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix} - \begin{bmatrix}0 & e_{31}\\ 0 & e_{33}\\ e_{15} & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix} - \begin{bmatrix}0 & h_{31}\\ 0 & h_{33}\\ h_{15} & 0\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix}D_x\\ D_y\end{Bmatrix} = \begin{bmatrix}0 & 0 & e_{15}\\ e_{31} & e_{33} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix} + \begin{bmatrix}\lambda_{11} & 0\\ 0 & \lambda_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix} + \begin{bmatrix}d_{11} & 0\\ 0 & d_{33}\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix}B_x\\ B_y\end{Bmatrix} = \begin{bmatrix}0 & 0 & h_{15}\\ h_{31} & h_{33} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix} + \begin{bmatrix}d_{11} & 0\\ 0 & d_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix} + \begin{bmatrix}\mu_{11} & 0\\ 0 & \mu_{33}\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\end{Bmatrix} \tag{4}$$

where $\sigma_{ij}$, $\varepsilon_{ij}$, $D_i$, $B_i$, $E_i$ and $H_i$ $(i, j = x, y)$ are components of stress, strain, electric displacement, magnetic induction, electric field and magnetic field, respectively.

The gradient equations are

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ E_i = -\phi_{,i},\ H_i = -\varphi_{,i}(i, j = x, y) \tag{5}$$

For an orthotropic, elastic half-space under in-plane loading, the constitutive equations and the equilibrium equations for plane strain are