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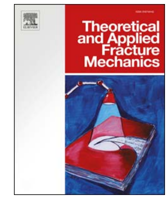

Three-dimensional analysis of an edge crack in a plate of finite thickness with the first-order plate theory

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ABSTRACT

In this paper, plasticity effects at the tip of an edge through-the-thickness crack are investigated by invoking the strip-yield model and the first order plate theory. The latter theory is the simplest extension of the classical plane theories of elasticity, which is capable of accounting for three-dimensional effects and, in particular, the plate thickness effect. By utilising the distributed dislocation technique, new results are obtained on the interaction of the free boundaries (free edge and free plate planes) with the crack tip plasticity region. Numerical results for the crack tip opening displacement and global constraint factor are first presented at various crack length to plate thickness and applied stress to yield stress ratios. The results are validated against past 3D solutions for an embedded crack as well as analytical solutions obtained under the plane stress or plane strain assumption. The new model and results can be utilised in advanced fracture and fatigue analyses of cracks emanating from a free surface.

1. Introduction

Crack tip plasticity has been widely investigated over the past fifty years. There have also been many attempts to develop simple analytical approaches to analyse plastic effects at the crack tip. A large portion of these approaches utilise the concept of the strip-yield model. This model, which was first proposed by Dugdale [1], assumes that near-tip yielding occurs in thin strip ahead of the crack tip. With this two-dimensional model, crack tip opening displacement (CTOD) and plasticity-induced crack closure (PICC) effects [2] can evaluated analytically for simple geometries and histories of loading. In a general case, this evaluation can be done in an incremental form, thus allowing to simulate the formation of the plastic wake and to obtain an estimation of the opening load and crack advance for each load cycle, which provides the way for assessment of the fatigue life of the structural component [3,4].

A significant advance in fracture and fatigue predictive tools over the past three decades was associated with the incorporation of the three-dimensional effects into crack and notch modelling [5–14]. It is well known that the evaluation of CTOD and PICC or other fracture and fatigue controlling parameters is significantly influenced by the out-of-plane constraint. Extensive discussion of the definition of the out-of-plane constraint and its influence on fracture controlling parameters was presented in [5,6,9–12]. It was shown analytically, numerically and in many experimental studies that the ratio of the plastic zone size over the plate thickness plays a key role in formation of the out-of-plane constraint and crack tip plasticity field. For a vanishingly small plastic zone around the crack tip, the stress state is dominated by the plane strain condition. With increase of the applied loads, i.e. increasing the plastic zone size, the stress fields develop towards the plane stress state [6].

In a modified strip-yield model, the three-dimensional effects are normally introduced via a single parameter, which is the global constraint factor, α_G. In accordance with this concept, the three-dimensional effects at a crack tip are simulated in a two dimensional strip-yield model by using factor α_G on tensile yielding; that is, the material yields when the stress is $>\alpha_G \times \sigma_Y$. The global constraint factor can be evaluated using the Finite Element (FE) method as an average through-the-thickness normal stress value over the crack tip plastic region [9]. From extensive FE simulations over the past three decades it was found the global constraint factor is rather independent of crack length and can be related to the applied stress-intensity factor for a given thickness [9]. In addition, the CTODs calculated from the modified strip-yield model agree well with the FE results from small- to large-scale yielding conditions for both thin and thick plates. The literature contains numerous examples of successful applications of the global constraint factor and the modified strip-yield model to prediction of fatigue life of plate components [9,12].

An alternative way to the FE simulation for the evaluation of the out-of-plane constraint or CTOD is analytical modelling. The simplest

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Nomenclature

Symbol	Description
x, y	in-plane Cartesian coordinates
z	out-of-plane Cartesian coordinate
u_x, u_y, u_z	displacement field components
h	plate half-thickness
N_{xx}, N_{yy}, N_{xy}	in-plane stress resultants
N_{zz}, R_x, R_y	out-of-plane stress resultants
λ, μ	Lamé parameters
E, ν	Young's modulus and Poisson's ratio
Φ	Airy's stress function
w	out-of-plane displacement function
κ	characteristic length-scale parameter, $\sim h^{-1}$
$\bar{\kappa}$	modified length-scale parameter, $\bar{\kappa}^2 = \kappa^2(1-\nu^2)$
$\hat{\sigma}_{xx}$, $\hat{\sigma}_{yy}$, $\hat{\sigma}_{xy}$	plane stress solution for in-plane stress components
w_{2D}	plane stress solution for the out-of-plane displacement function
w_h	decaying solution for the out-of-plane displacement function
$\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$	principal stress components
a	length of edge crack (or half-length of embedded crack)
ω	plastic strip length
σ_Y	yield strength
σ_{ap}	applied crack opening stress
α_G	global plastic constraint factor
K_I	mode I stress intensity factor
B_y	distributed dislocation density function
K_{yy}, K_{zz}	Cauchy kernels describing normal tractions generated by distributed dislocations in the y and z directions, respectively.
t_k,s_i	integration and collocation points for the numerical solution of the governing Singular Integral Equation (SIE)
N	number of integration points

theory which is capable to evaluate the 3D stress field is the first order plate theory suggested by Kane and Mindlin in 1956 [15]. This theory was originally applied to the analysis of high-frequency extensional vibrations in moderately thick plates. It assumes that the out-of-plane deformations are uniform across the plate thickness. The governing equations of this theory incorporate all six independent stress components, while retain the simplicity of a two-dimensional formulation [15,16]. This theory was used previously by several researchers for the elastic three-dimensional analysis of crack problems [17–19]. However, it seems, Kotousov [20–22] was the first to apply this theory to evaluate the plasticity effects, constraint factor and CTOD for through-the-thickness cracks in plates of finite thickness. This theory was also applied by Kotousov and his collaborators for the evaluation of PICC, effects of the plate thickness and overload on fracture and fatigue crack growth rates [23–26]. However, the application of this theory is still limited to relatively simple geometries, in particular, embedded cracks in infinite plate geometries.

The current paper will address this shortcoming and examine a more realistic geometry i.e. an edge crack in a finite thickness plate within the framework of the first order plate theory and examine plasticity effects using the strip-yield model. Recently the present authors developed a new simplified approach for the analysis of 3D problems of plane elasticity with the first order plate theory [27]. It was demonstrated that this approach works well for singular and non-singular problems of plane elasticity [27,28]. This simplified approach will form the basis for the analytical solutions developed here. The main outcomes of the present study are in-depth results for the dependence of the CTOD, plastic zone size, and plastic constraint factor upon the plate thickness, crack length and applied crack opening stress as well as a direct comparison of the embedded and edge crack configurations.

In Section 2, the governing equations of the first order plate theory are summarised. Then, in Section 3, a brief description of the new simplified approach for obtaining approximate solutions within the first order plate theory is provided. In Section 4, a solution for an edge dislocation in a semi-infinite plate of finite thickness is obtained and in Section 5, this solution is utilised to formulate the governing equation for an edge crack in a finite thickness plate using the distributed dislocation density technique. In Section 6, this new solution is compared with previous 3D FE results for an embedded crack as well as with analytical solutions for edge cracks developed with plane stress assumptions. Finally, new 3D results for an edge crack and comparison against the embedded crack configuration are presented in Section 7.

2. Governing equations of the first order plate theory

The equations presented here follow from Yang and Freund [17], with the exception that the shear correction factor appearing in the original equations of Yang and Freund [17] is set to unity. This way, the special case of plane stress state can be recovered exactly from the presented equations for the first order plate theory. In accordance with the kinematic assumption of this theory [15], the displacement field in an elastic plate bounded by planes $z= \pm h$ can be approximated by the following equations:

$$u_x = u_x(x,y), \quad u_y = u_y(x,y), \quad u_z = \frac{z}{h}w(x,y), \tag{1}$$

where $w(x,y)$ is the out-of-plane displacement of the surface $z= h$ of the plate. The stress resultants are given by

$$(N_{xx},N_{yy},N_{zz},N_{xy}) = \int_{-h}^{h} (\sigma_{xx},\sigma_{yy},\sigma_{zz},\tau_{xy})dz, \tag{2}$$

and

$$(R_x,R_y) = \int_{-h}^{h} (\tau_{zx},\tau_{zy})zdz. \tag{3}$$

Making use of Hooke's law, the in-plane and transverse shear stress resultants can be expressed in terms of the displacement components, as

$$\frac{N_{\alpha\beta}}{2h} = \lambda\theta\delta_{\alpha\beta} + \mu(u_{\alpha,\beta} + u_{\beta,\alpha}), \tag{4a}$$

$$\frac{N_{zz}}{2h} = \lambda\theta + 2\mu\frac{w}{h}, \tag{4b}$$

$$\frac{R_\alpha}{2h} = \frac{h}{3}\mu\frac{\partial w}{\partial \alpha}, \tag{4c}$$

where the indices $(\alpha,\beta) = (x,y)$, and the comma denotes partial differentiation with respect to the coordinate. The parameter θ denotes the volumetric strain,

$$\theta = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{w}{h}, \tag{5}$$

and λ and μ are the Lamè constants, which are related to the Young's modulus E and Poisson's ratio ν via the following expressions:

$$\lambda = \frac{\nu E}{(1 + \nu)(1-2\nu)}, \tag{6a}$$

$$\mu = \frac{E}{2(1 + \nu)}. \tag{6b}$$

The three equilibrium equations, in the absence of body forces and

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