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Analysis of arbitrarily shaped, planar cracks in a three-dimensional transversely isotropic thermoporoelastic medium

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ABSTRACT

The displacement discontinuity boundary integral equation method is extended to analyze planar cracks of arbitrary shape embedded in a three-dimensional, transversely isotropic, thermoporoelastic medium. Based on the general solutions and Hankel transform technique, the fundamental solutions for unit-point, extended displacement discontinuities (including the displacement discontinuities, pore pressure discontinuity, and the temperature discontinuity) are derived. The extended displacement discontinuity boundary integral equations are established for an arbitrarily shaped, planar crack in the isotropic plane of the thermoporoelastic medium in terms of the extended displacement discontinuities. Using the boundary integral equations method, the singularities of near-crack front fields are analyzed, and the stress, fluid flux and heat flux intensity factors are derived in terms of the extended displacement discontinuities. To validate the analytical solution, the EDD boundary element method is proposed. The numerical simulation of a penny-shaped crack under combined uniform mechanical-pore pressure–thermal loadings is compared with the analytical solution to validate the correctness of the proposed method. As an application, two coplanar elliptical cracks are numerically simulated. The influences of the applied, combined mechanical-pore-pressure-thermal loadings, the crack distance, the ellipticity ratio as well as the size of cracks are all studied.

1. Introduction

Due to their specific features of high acoustic and thermal insulation, specific surface area and excellent penetrability, porous materials have become one of the fastest-growing materials in the modern engineering application, and they have been widely used in foodstuff, chemical industry, architectural engineering and energy industry and so on. There commonly exist defects such as cracks, inclusions and cones in the manufacturing and production of this material, it is essential to understand the fracture mechanisms and set up the corresponding fracture criteria.

Biot [1] originally set up the consolidation model based on elastic theory and fundamental equations of porous media and proposed a general theory of three-dimensional consolidation. Following that, Biot [2] established the equations of elasticity and consolidation for a porous elastic material containing a fluid. To take the temperature effect into consideration for engineering applications like heat supply pipeline, nuclear storage facilities and deep drilling, the thermoporoelasticity model was founded [3,4]. Based on the basic relations, many researchers derived the Green's functions for transversely isotropic, poroelastic materials. Taguchi and Jurashige [5] presented the Green's functions for an infinite, fluid-saturated, transversely isotropic, poroelastic material by virtue of the Kupradze's method combined with the triple Fourier transforms and Hankel transforms. Li et al. [6] obtained the steady-state general solution for transversely isotropic, thermoporoelastic media using the potential function method. Furthermore, Hou et al. [7] derived the Green's functions for 3D, transversely isotropic thermoporoelastic bi-materials. Later on, Lu et al. [8] analyzed a thermoporoelastic beam by utilizing the general solution of thermoporoelasticity and the Lur'e method. Wu et al. [9] analyzed the steady-state deformation of axisymmetric, transversely isotropic, thermoporoelastic circular cylinders. Kumar et al. [10,11] studied the deformation of a half space with incompressible fluid as a result of inclined load of arbitrary orientation and the response of thermomechanical sources in a thermoporoelastic medium, respectively. Wu et al. [12] gave the Green's functions for axisymmetric cones under the action of point fluid source and heat source in thermoporoelastic media.

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As the crack problem is a hotspot in thermoporoelasticity, many researches have been conducted. Atkinson and Craster [13] studied the plain strain fracture problems of poroelasticity. Gordeyev [14] analyzed a disk-shaped crack in a transversely isotropic, fluid-saturated, poroelastic material. Adelaide et al. [15] gave the analytical solution of hydrostatic-elastic problem considering the interaction of wetting fluid with a penny-shaped circular crack. Li et al. [16] analyzed a penny-shaped crack under combined thermo-poro-mechanical loads in a thermoporoelastic body on the basis of potential function method and the general solutions [6]. It can be concluded that most of the previous literature was limited to two-dimensional (2D) crack problems or some 3D cracks of certain shapes under uniformly distributed loadings. However, the crack usually displays various shapes and the applied loads are also complicated. Therefore, it is necessary to propose an analysis method for an arbitrarily shaped crack under different kinds of combined loading in thermoporoelastic media.

There are many methods proposed to deal with crack problems in porous media, such as finite difference method, finite element method, discrete element method, and finite volume method, etc. In addition, Sladek et al. [17] employed the scaled boundary-finite element method (SBFEM) to analyze cracks in porous piezoelectric solids. Goudarzi and Mohammadi [18] used the extrinsically enriched Element-free Galerkin (EFG) approach to analyze cohesive cracking in saturated porous media. Yan and Zheng [19] utilized a combined finite-discrete element method (FDEM) to study hydraulic fracture in porous media. Phurkhao used the numerical Laplace inversion technique to investigate the dynamic stress intensity factors of an in-plane shear crack in saturated porous medium [20] and then extended it to analyze the transient response of a saturated porous cylinder containing a penny-shaped crack subjected to a suddenly applied normal loading [21]. Comparing with the methods mentioned above, the boundary element method distinguishes itself as a boundary method, namely, the numerical discretization is conducted at reduced spatial dimension [22]. The displacement discontinuity method (DDM) proposed by Crouch [23] treats the crack as one surface so that the discretization is required on only one side of the crack. This method was initially intended to investigate 2D elastic crack problems. Later on, researchers found this method widely applicable and efficient in analyzing crack problems in other materials. Later on, this method was extended to study 3D, elastic media [24,25], piezoelectric media [26], magnetoelectroelastic media [27], and thermoelastic media [28], where the original elastic displacement discontinuities were extended to include the temperature, electric potential, and magnetic potential discontinuities across crack faces.

Due to the existence of cracks, the pore pressure and temperature distribution across crack faces are discontinuous for thermoporoelastic media. Motivated by this, this paper develops the extended displacement discontinuity, boundary hyper-singular integral equation method to analyze an arbitrarily shaped, planar crack in 3D, transversely isotropic thermoporoelastic media. The paper is organized as follows: Section 2 lists the basic equations, and the fundamental solutions for unit-point EDDs are obtained in Section 3. The boundary hyper-singular integral equations for arbitrarily shaped, planar cracks are established in Section 4. In Section 5, the singular behavior near the crack front is analyzed, and the extended field intensity factors are obtained in terms of the EDDs. In Section 6, the numerical method for two coplanar elliptical cracks is proposed and the algebraic equations are presented. In Section 7, the correctness of the numerical method is verified, and the extended stress intensity factors are illustrated at various crack distance, the ellipticity ratio as well as the size of the cracks. At last, conclusions are drawn in Section 8.

2. Basic equations

The constitutive relations for transversely isotropic thermoporoelastic media, referred to the Cartesian coordinates (x,y,z) with xoy coincident with the isotropic plane and the z-axis identical to the axis of rotational material symmetry, can be expressed as [3,4]

$$\begin{aligned}
\sigma_x &= c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \alpha_1 P - \beta_1\theta,\\
\sigma_y &= c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} - \alpha_1 P - \beta_1\theta,\\
\sigma_z &= c_{13}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + c_{33}\frac{\partial w}{\partial z} - \alpha_3 P - \beta_3\theta,
\end{aligned} \tag{1a}$$

$$\begin{aligned}
&\sigma_{xy} = c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \sigma_{yz} = c_{44}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right), \sigma_{zx} = c_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right),\\
&P = M\left[\zeta - \alpha_1\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \alpha_3\frac{\partial w}{\partial z} + \beta_m\theta\right],
\end{aligned} \tag{1b}$$

where u, v, and w are displacements and σ_{ij} is stress; P and θ are changes of the pore pressure and temperature, respectively; $P = 0$ and $\theta = 0$ correspond to the free stress state; ζ is the variation of fluid content; c_{ij}, α_1 (α_3, M) and β_1 (β_3, β_m) are elastic moduli, Biot's effective stress coefficients and thermal constants, respectively, where we have a reciprocal relation $2c_{66} = c_{11} - c_{12}$.

Fluid flow and heat conduction in porous media follow the Darcy law [29] and Fourier law, respectively

$$\begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} = -\begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{11} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix}\begin{Bmatrix} P_{,x} \\ P_{,y} \\ P_{,z} \end{Bmatrix}, \quad \begin{Bmatrix} h_x \\ h_y \\ h_z \end{Bmatrix} = -\begin{bmatrix} \lambda_{11} & 0 & 0 \\ 0 & \lambda_{11} & 0 \\ 0 & 0 & \lambda_{33} \end{bmatrix}\begin{Bmatrix} \theta_{,x} \\ \theta_{,y} \\ \theta_{,z} \end{Bmatrix}, \tag{1c}$$

where q_x, q_y and q_z are the fluid fluxes; h_x, h_y and h_z are the heat fluxes; $\kappa_{11}(\kappa_{33})$ and $\lambda_{11}(\lambda_{33})$ are coefficients of permeability and thermal conductivity, respectively.

According to Li et al. [6], the thermoporoelastic loadings are assumed to vary slowly with the rates of fluid mass content and entropy vanishing. Accordingly, in a steady-state case, the pore pressure and temperature fields remain constant and are governed, respectively, by the following two Laplace equations

$$\left(\kappa_{11}\frac{\partial^2}{\partial x^2} + \kappa_{11}\frac{\partial^2}{\partial y^2} + \kappa_{33}\frac{\partial^2}{\partial z^2}\right)P = 0, \tag{2a}$$

$$\left(\lambda_{11}\frac{\partial^2}{\partial x^2} + \lambda_{11}\frac{\partial^2}{\partial y^2} + \lambda_{33}\frac{\partial^2}{\partial z^2}\right)\theta = 0. \tag{2b}$$

It is noted that the one-way coupling theory for thermoporoelastic media is adopted in the paper, which means that, the pore pressure and thermal loadings influence the elastic fields, while the mechanical loadings have no impact on the thermal and hydraulic fields. In addition, the thermal and hydraulic fields are independent and do not interfere with each other.

In the absence of body forces, the mechanical equilibrium equations can be expressed as

$$\begin{aligned}
&\frac{\partial\sigma_x}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{zx}}{\partial z} = 0,\\
&\frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\sigma_{yz}}{\partial z} = 0,\\
&\frac{\partial\sigma_{zx}}{\partial x} + \frac{\partial\sigma_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0.
\end{aligned} \tag{3}$$

3. Fundamental solutions for unit point EDDs

Assuming a penny-shaped crack with radius a centered at the origin of the coordinate system is oriented in the isotropic plane xoy perpendicular to the rotational material symmetry direction, as shown in Fig. 1. The upper and lower surfaces of the crack are denoted by S^+ and S^-, respectively, and

$$x = r\cos\phi, y = r\sin\phi. \tag{4}$$

The extended displacements across the crack faces are discontinuous, and the extended displacement discontinuities (EDDs) can be expressed as