Theoretical and Applied Fracture Mechanics xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Theoretical and Applied Fracture Mechanics

journal homepage: www.elsevier.com/locate/tafmec

A new coupled method for high-accuracy determination of fracture parameters of an interface V-notch in magneto-electro-elastic bimaterial

Chenghui Xu [a,b], Wang Xu [a], Dalun Rong [a], Zhenhuan Zhou [a,*], Andrew Y.T. Leung [c], Xinsheng Xu [a]

[a] State Key Laboratory of Structure Analysis of Industrial Equipment, Department of Engineering Mechanics, International Research Center for Computational Mechanics, Dalian University of Technology, Dalian 116024, PR China
[b] School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an 710072, PR China
[c] School of Engineering and Applied Science, Aston University, UK

ARTICLE INFO

Article history:
Received 5 January 2017
Revised 13 April 2017
Accepted 21 April 2017
Available online xxxx

Keywords:
Symplectic method
V-notched magneto-electro-elastic composites
Interface fracture
Intensity factor
Energy release rate

ABSTRACT

A simple coupled method, called "finite element discretized symplectic method" is introduced into the mode III fracture analysis of a V-notched magneto-electro-elastic (MEE) bimaterial. High-accuracy generalized intensity factors and energy release rate are computed to evaluate the singularities of mechanical, electric and magnetic fields. The present method is carried out in two steps. In the first step, the physical domain is meshed by the conventional element for MEE media and is divided into a finite size singular region near the notch tip and a regular region far away from the notch tip. In the second step, analytical symplectic eigenfunctions are employed to transform the large number of nodal unknowns in the singular region into a small set of undetermined coefficients of a symplectic series; the nodal unknowns in the regular region remain as usual. Consequently, the computational cost is enormously reduced and fracture parameters are actually the first three coefficients of the series. Explicit expressions in the singular fields are obtained simultaneously. Numerical results are compared with the existing solutions and found to be in good agreement. Some new results are given also.

1. Introduction

Magneto-electro-elastic (MEE) composites are a new class of smart materials which are consisted of piezoelectric and piezomagnetic phases. In contrast to the single-phase materials, MEE composites exhibit relatively stronger coupling effects among mechanical, electric and/or magnetic fields [1]. Therefore, such composites have played a great important role in the development of novel multifunctional devices, smart electronic structures such as magnetic/electric transducers, actuators, and sensors [2]. Various topics concerning the MEE composites have been widely studied by numerous scholars [3–9]. Like in conventional composites, defects or flaws are inevitably introduced during the manufacturing process or during the service by impact loading. The existences of defects would lead to the concentration of stress, electric displacement and magnetic induction and may further cause disruption. Therefore, a better understanding on the fracture behavior of MEE composites will play an important role for the optimum design and reliable service performance.

In recent years, the cracks arisen in MEE composites have been extensively investigated by many researchers. The intensity factors of the mechanical, electric and magnetic fields are derived by various theoretical methods, such as Schmidt method [10,11], integral transform method [12–17], complex variable method [18–21], method of analytical continuation [22], new energy method [23]. However, most of the above-mentioned literatures are concentrated on the infinite-size MEE composites. Only Refs. [14–16] referred to the finite-size bodies. In practice, MEE components usually involved finite geometries with irregular shapes, complexity of geometric cracks and loading condition. Therefore, many numerical methods are developed to analyze the fracture of MEE composites, such as finite element method (FEM) [24–28], boundary element method (BEM) [29–32] and meshless method [33,34]. Beside the crack, the interface V-shape notch is also one of the most important defects in MEE composites, which greatly reduces the fracture resistance or shorten the service life of MEE devices. Compared to literatures on cracks, only a few studies were carried out on the V-notch in MEE composites [35–38]. The singularities of the V-notch are reported, instead of analytical solutions near the

* Corresponding author at: State Key Laboratory of Structure Analysis of Industrial Equipment, Department of Engineering Mechanics, International Center for Computational Mechanics, Dalian University of Technology, Dalian 116024, PR China.

E-mail address: zhouzh@dlut.edu.cn (Z. Zhou).

http://dx.doi.org/10.1016/j.tafmec.2017.04.016
0167-8442/

notch tip or fracture parameters for evaluating the fracture behaviors.

In view of the open literatures, it is found that the analytical analysis of MEE composites with interface V-notches are very limited, especially the fracture parameters. Although numerical approaches could deal with the notched MEE structures, no exact solutions around the notch tip can be founded. Motivated by these considerations, this paper presents a finite element discretized symplectic method (FEDSM) to evaluate the interface V-notches and obtain explicit solutions at the notch tip. The FEDSM for MEE composites is developed based on the authors previous work on fracture analysis of elastic and piezoelectric media [39,40]. The MEE composite is divided into two regions: a finite size singular stress region near the notch tip (near field) and a regular region far away from the notch tip (far field). The overall body is modeled by the conventional FEM. Using the symplectic method proposed by Zhong and his collaborators [41–44], the near field is enhanced by analytical symplectic eigenfunctions [45,46]. With the aid of the eigenfunctions, the large number of nodal unknowns in the near field (z-displacements, electric potentials and magnetic potentials) are reduced to a small set of undetermined coefficients of the symplectic expansion while the nodal unknowns in the far field remain unchanged. Explicit expressions of the near field and fracture parameters of the notch are directly computed without any-post processing.

This paper is outlined as follows. The basic equations and finite element formulation for MEE bimaterial are presented in Section 2. The Hamiltonian system is established and solved in Section 3, and the definitions of fracture parameters are described in Section 4. In Section 5, the formulation of FEDSM for MEE bimaterial is addressed. Numerical results and discussions on the accuracy and efficiency are shown in Section 6. There is a conclusion in the last section.

2. Fundamental problem and finite element formulation for MEE composites

Consider a MEE bimaterial with an interface V-type notch, as shown in Fig. 1. The upper and lower material elements are denoted by M_1 and M_2, respectively. α is the distribution angle. Both Cartesian coordinates (x, y, z) and cylindrical coordinates (r, θ, z) are selected, where z-axis is oriented in the poling direction of the MEE bimaterial. The overall body is divided into two regions, namely, near field (Ω_N) and far fields (Ω_F). The curve Γ_0 denotes the boundary between the regions.

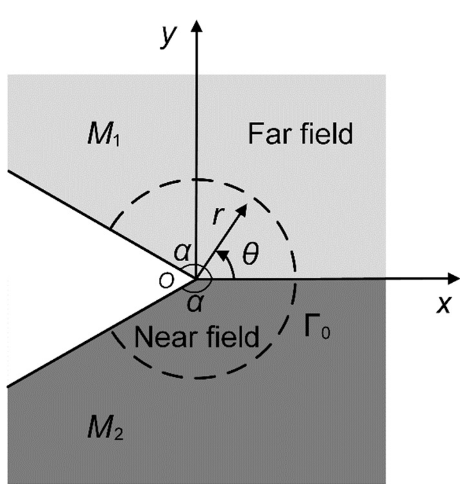

Fig. 1. A V-notched MEE bimaterial.

2.1. Basic equations in Cartesian coordinates

For the anti-plane fracture problem, the basic equations written in Cartesian coordinates (x, y, z) are expressed as follows:

Constitutive equations:

$$\{\sigma_{kz}^{(i)},\ D_k^{(i)},\ B_k^{(i)}\}^{\mathrm{T}} = \mathbf{M}^{(i)}\{\gamma_{kz}^{(i)},\ E_k^{(i)},\ H_k^{(i)}\}^{\mathrm{T}}(k = x,\ y); \tag{1}$$

Linear geometric equations:

$$\gamma_{kz}^{(i)} = \partial_k w^{(i)}, E_k^{(i)} = -\partial_k \phi^{(i)}\ , H_k^{(i)} = -\partial_k \varphi^{(i)}(k = x,\ y); \tag{2}$$

Equilibrium equations:

$$\begin{cases} \partial_x \sigma_{xz}^{(i)} + \partial_y \sigma_{yz}^{(i)} + F^{(i)} = 0 \\ \partial_x D_x^{(i)} + \partial_y D_y^{(i)} + Q^{(i)} = 0 \\ \partial_x B_x^{(i)} + \partial_y B_y^{(i)} + I^{(i)} = 0 \end{cases} \tag{3}$$

where $\mathbf{M}^{(i)} = \begin{bmatrix} C_{44}^{(i)} & -e_{15}^{(i)} & -h_{15}^{(i)} \\ e_{15}^{(i)} & \kappa_{11}^{(i)} & g_{11}^{(i)} \\ h_{15}^{(i)} & g_{11}^{(i)} & \chi_{11}^{(i)} \end{bmatrix}$ is the matrix of material properties; the superscript i ($i = 1,\ 2$) represents the number of material; ∂_k denotes partial differentiation to k; $w^{(i)}$, $\phi^{(i)}$ and $\varphi^{(i)}$ are anti-plane displacement, electric potential and magnetic potential, respectively; $\sigma_{kz}^{(i)}$, $\gamma_{kz}^{(i)}$, $D_k^{(i)}$, $E_k^{(i)}$, $B_k^{(i)}$ and $H_k^{(i)}$ are components of stress, strain, electrical displacement, electrical field, magnetic induction and magnetic field, respectively; $C_{44}^{(i)}$, $e_{15}^{(i)}$, $h_{15}^{(i)}$ and $g_{11}^{(i)}$ are the elastic, piezoelectric, piezo-magnetic and electromagnetic constants, respectively; $\kappa_{11}^{(i)}$ and $\chi_{11}^{(i)}$ are the dielectric permittivity and magnetic permeability coefficients, respectively; $F^{(i)}$, $Q^{(i)}$ and $I^{(i)}$ are the body force, electric charge density and electric current density, respectively.

2.2. Finite element formulation

Eight-node quadrangle element is used here. For MEE composites, the fundamental unknowns of a node are three components $w_j^{(i)}$, $\phi_j^{(i)}$ and $\varphi_j^{(i)}$, where the subscript j represents the number of nodes. The fundamental unknowns within an element can be represented as follows

$$w^{(i)} = \sum_{j=1}^{8} N_j w_j^{(i)}, \phi^{(i)} = \sum_{j=1}^{8} N_j \phi_j^{(i)}, \varphi^{(i)} = \sum_{j=1}^{8} N_j \varphi_j^{(i)} \tag{4}$$

in which $\mathbf{N} = \{N_1,\ \cdots,\ N_8\}^{\mathrm{T}}$ is the interpolation function. Defining displacement vectors as

$$\bar{\mathbf{u}}_e^{(i)} = \{w_1^{(i)},\ \cdots,\ w_8^{(i)}\}^{\mathrm{T}}, \tilde{\mathbf{u}}_e^{(i)} = \{\phi_1^{(i)},\ \cdots,\ \phi_8^{(i)}\}^{\mathrm{T}}, \hat{\mathbf{u}}_e^{(i)} = \{\varphi_1^{(i)},\ \cdots,\ \varphi_8^{(i)}\}^{\mathrm{T}}. \tag{5}$$

Substituting Eq. (5) into Eq. (2), the strain vectors can be obtained

$$\begin{cases} \boldsymbol{\gamma}^{(i)} = \{\gamma_{xz}^{(i)},\ \gamma_{yz}^{(i)}\}^{\mathrm{T}} = \bar{\mathbf{B}}\bar{\mathbf{u}}_e^{(i)} \\ \mathbf{E}^{(i)} = \{-E_x^{(i)},\ -E_y^{(i)}\}^{\mathrm{T}} = \tilde{\mathbf{B}}\tilde{\mathbf{u}}_e^{(i)} \\ \mathbf{H}^{(i)} = \{-H_x^{(i)},\ -H_y^{(i)}\}^{\mathrm{T}} = \hat{\mathbf{B}}\hat{\mathbf{u}}_e^{(i)} \end{cases} \tag{6}$$

where $\bar{\mathbf{B}} = -\tilde{\mathbf{B}} = -\hat{\mathbf{B}} = \mathbf{J}^{-1}\begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \cdots & \frac{\partial N_8}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_8}{\partial \eta} \end{bmatrix}$ and $\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$.

The potential energy in the field $V^{(i)}$ with the boundary can be expressed as

$$\Pi^{(i)} = \int_{V^{(i)}} \Psi^{(i)} dV - \int_{V^{(i)}} (F^{(i)} w^{(i)} + Q^{(i)} \phi^{(i)} + I^{(i)} \varphi^{(i)}) dV \tag{7}$$

Please cite this article in press as: C. Xu et al., A new coupled method for high-accuracy determination of fracture parameters of an interface V-notch in magneto-electro-elastic bimaterial, Theor. Appl. Fract. Mech. (2017), http://dx.doi.org/10.1016/j.tafmec.2017.04.016