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A strain energy method for elastic–plastic analysis of notches under shear loading

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ABSTRACT

Neuber's rule and the equivalent strain energy density (ESED) method are two well-known approximate approaches for elastic–plastic notch analysis. Neuber's rule often overestimates whereas the ESED method underestimates the values of notch root stresses and strains. In the present study an approximate method has been proposed, which is based on the strain energy density analysis for notches under shear loading. The proposed method has been assessed against the FEA results from the literature and a good agreement has been observed. Besides predicting notch strains for loading with predominant plastic zone, a significant improvement is found in contrast with those estimated by the Neuber's rule or the ESED method.

1. Introduction

Fracture of components usually initiates at the stress concentrations where local stresses and strains are higher than nominal ones. Therefore, it is important to estimate accurately the local elastic–plastic stresses and strains at the notch-root. Elastic–plastic finite element analysis (FEA) is an accurate method, however it is an expensive and time consuming technique. Hence, approximate methods have been put forward in order to estimate the notch-root strain and stress concentrations. One of the widely used approximate approaches is Neuber's rule proposed in 1961 [1],

$$K_\tau K_\gamma = K_t^2 \tag{1}$$

where K_t is an elastic stress concentration factor and $K_\tau = \tau/\tau_{nom}$ and $K_\gamma = \gamma/\gamma_{nom}$ are the stress and strain concentration factors, respectively.

It can be noted that Neuber originally derived his rule for out-of-plane shear loading and his rule in the form of Eq. (1) is routinely used also for other loadings such as tension/compression or bending.

Substituting the relations for K_τ and K_γ, into Eq. (1) and after arrangement one gets

$$\tau\gamma = K_t^2 \tau_{nom}\gamma_{nom} \tag{2a}$$

or

$$\frac{1}{2}\tau\gamma = K_t^2 \frac{\tau_{nom}\gamma_{nom}}{2} \tag{2b}$$

For nominal stresses being smaller than the yield strength, $\tau_{nom} < \tau_Y$, the right hand side (RHS) of Eq. (2b) represents the nominal elastic strain energy density multiplied by K_t^2. A graphical interpretation of the left hand sides (LHSs) of Eqs. (2a) and (2b) are shown in Fig. 1.

It is interesting to note that Neuber proposed a several different relationships (see Reference [2]), from which Eq. (1) is the most widely used. As it is pointed out in [2], a number of expressions derived by Neuber over the years can be unified by the following relationship

$$K_\tau K_\gamma = \Psi K_t^2 \tag{3}$$

where the unified parameter ψ is equal to:

$\Psi = 0.5$ for the law published in 1958,
$\Psi = 1$ for the law published in 1961,
$\Psi = 1 - 0.1/K_t + K_\tau/K_t^2 - 0.9\, K_\gamma/K_t^2$ for the law published in 1985,
$\Psi = 1 - 1/K_t + K_\tau/K_t^2$ for the law published in 1985.

The further details and discussions regarding the parameter ψ can be find out in Reference [2].

In the recent years, Zappalorto and Lazzarin [3–6] carried out extensive investigations on the stress and strain fields close to notches utilizing different forms of stress–strain laws. Their results can be also represented by Eq. (3) [2].

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Nomenclature

K_t	elastic stress concentration factor	γ_P	plastic shear strain at notch
K_τ	stress concentration factor	n	strain hardening exponent
K_γ	strain concentration factor	G	shear modulus of elasticity
τ	shear stress at notch	G_S	secant shear modulus
τ_{nom}	nominal shear stress	ESED	equivalent strain energy density
τ_Y	yield shear stress	FEA	finite element analysis
γ	shear strain at notch	KS	Kujawski–Salemi method
γ_{nom}	nominal shear strain	LHS	left hand side
$\gamma_{elastic}$	elastic shear strain	N	Neuber's rule
γ_{Pnom}	nominal plastic shear strain	RHS	right hand side

Another approach often used for elastic–plastic analysis at notch is the equivalent strain energy density (ESED) method proposed by Molski and Glinka [7] in 1981 originally formulated for tensile stresses and strains. Since this paper considers pure shear loading, Eq. (4) represents the ESED method in terms of the shear stress and strain.

$$\int \tau\, d\gamma = K_t^2 \frac{\tau_{nom}\gamma_{nom}}{2} \tag{4}$$

A graphical interpretation of the LHS of Eq. (4) is given in Fig. 2. Fig. 3 illustrates graphically the comparison between the LHSs of Eqs. (2b) and (4). An examination of Fig. 3 indicates that in order to satisfy the RHS of Eqs. (2b) or (4), which are the same, the LHS of Eq. (2b) would result in larger values of the notch stresses and strains than the LHS of Eq. (4). Moftakhar et al. [8] advocated that Neuber's rule could be considered as the upper limit whereas the ESED method as the lower limit for the actual elastic–plastic behavior of the notch strain.

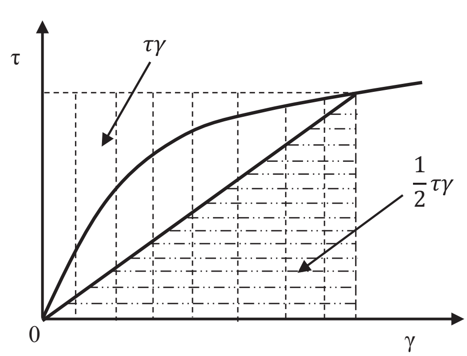

Fig. 1. Graphical illustration of the left hand side of Eqs. (2a) and (2b).

Recently, Kilambi and Tipton [9] reported elastic–plastic finite element analysis (FEA) simulations for out-of-plane shear loading, analogous to that used by Neuber, considering three different notch geometries and three strain hardening exponents. It is shown that FEA results are between Neuber's and ESED estimations. In addition, Kilambi and Tipton [9] demonstrated that the success of Neuber's rule depends on the material strain hardening exponent. Especially, Neuber's rule performed reasonably well when the strain hardening exponent, n, for the Ramberg–Osgood material model is equal 0.2. The Ramberg–Osgood relationship with $n = 0.2$ exhibits a similar stress–strain behavior as Neuber's material model used in his 1961 analysis [1]. For lower values of the strain hardening exponent of 0.1 and 0.05 the agreement between FEA results and estimates from Neuber's rule diminished in particular for the lowest value of n equal to 0.05.

It can be noted that approximate methods for multiaxial stress and strain analysis at notch have been proposed to study fatigue damage under multiaxial loading, e.g. [10,11]. Recently, Ince et al. [12] have developed a computational model to estimate multiaxial elastic–plastic notch stress and strain by using linear elastic FE stresses, for notch components subjected to non-proportional multiaxial loadings. Comparing strain histories results against experimental data shows a reasonable accuracy for six different non-proportional paths. Ince [13] has also compared the predicted local stress and strain from the developed computational model with the elastic plastic FEA results for both monotonic and cyclic non-proportional loadings. The author stated that the experimental or elastic plastic FEA data might be substituted with the proposed model results to predict fatigue life only in the early stage of design analysis. Marangon et al. [14] and Campagnolo et al.

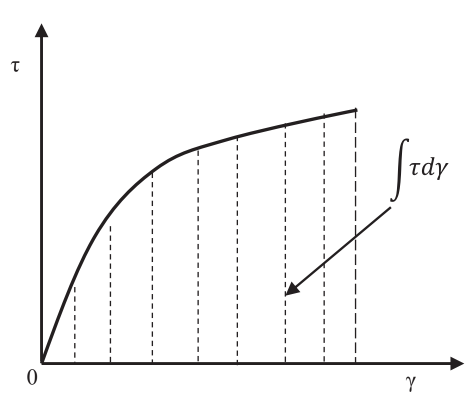

Fig. 2. Graphical illustration of the left hand side of Eq. (4).

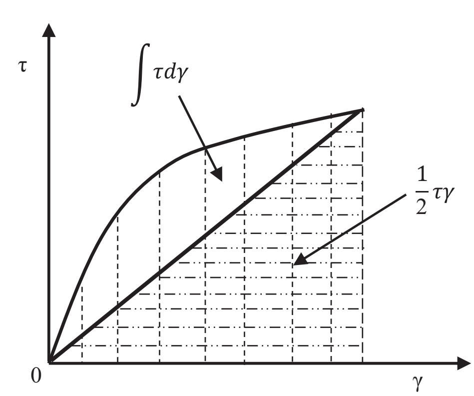

Fig. 3. Graphical illustration of the left hand side of Eqs. (2b) and (4).

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