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Stress intensity factors from stress analysis of an equivalent hole

Paolo Livieri

Department of Engineering, University of Ferrara, via Saragat 1, 44122 Ferrara, Italy

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ABSTRACT

In this paper, on the basis of the stress analysis of an equivalent model, the stress intensity factors (SIF) of a crack have been evaluated. In the stress analysis, the crack is substituted by an equivalent hole equal in size to the length of the crack. The method is based on the evaluation of the hoop stress on the free border of the equivalent hole and the subsequent calculation of the *J*-integral ($J_{V\rho}$) as a parameter related to the SIF. Alternatively, if the hoop stress on the free border cannot be evaluated, the stress analysis can be performed on the surface in the neighbourhood of the hole.

In order to validate the method, two experimental cases have been considered: a plate with two different bore diameters of 50% and 20% of the plate width, respectively. The stress analysis was performed by means of strain gauges attached to the free border. For the plate with the smaller bore, the strain gauges were also attached to the surface near the hole. In this way, the strain approach can be extended to thin plates or when it is not possible to attach the strain gauges to the free border of the hole.

1. Introduction

The measurement of some physical quantities related to the displacement field or to the strain field in the neighbourhood of the crack tip allows the researcher to calculate the stress intensity factor (SIF). The direct evaluation of SIF from experimental analysis requires the use of optical techniques [1,2] or accurate measurements of the temperature variation [3]. Due to the small size of the area where the high stress gradient occurs, the measurements must be very localised and, if possible, also be very accurate. The use of a common strain gauge with traditional grids (length of some millimetres), is not normally able to capture the local effect due to a singular stress field. On the other hand, the use of a strain gauge gives an engineer and a reliable response of stress measurements [4]. The application of a strain gauge with traditional grids is used, for example, in the field of welded joint integrity for the evaluation of structural stress [5–7]. The local stress field needed for fatigue assessments is extrapolated by means of the experimental strain measurements made in two or three points near the weld toe. However, the extrapolation at the weld toe is a difficult task because the trend of stress is the sum of the singular symmetric and skew-symmetric stress field related to Williams eigenvalue [8,9] and many choices are possible [6,7].

As far as the SIF assessments are concerned, in order to avoid an analysis of a singular stress field, the crack could be replaced by an elliptical notch of the same size. This idea was proposed by Irwin in Ref. [10] and was supported by Neuber's results [11]. Under mode I loadings, Irwin verified that the product of peak stress σ_{max} by the square root of the notch tip radius ρ gave the SIF when ρ converges to zero. Unfortunately, for a mode II loading, the stress at the notch tip is null and this gives some problems when assessing the mode II SIF (see for instance Refs. [12,13]). So that, the peak stress at the notch tip, as reported by Sih and Liebowitz [14], can be replaced by the maximum hoop stress due to the mode II loading. Alternatively, as shown in Ref. [15], the *J*-integral may be helpful to analyse the singularity of the stress field for cracks as introduced by Rice [16]. For elliptical, parabolic and hyperbolic notches as well as V-sharp notches, the use of the classical *J*-integral was generalised in Refs. [17,12]. In these cases, the *J*-integral is not strictly a path-independent integral [18]. To ensure clarity, the *J*-integral applied to a generic notch is indicated as $J_{V\rho}$, whereas the symbol J is used for the classical *J*-integral evaluated for a crack. In order to improve accuracy in the use of an ellipse as an equivalent notch, in Ref. [19] the component of *J*-integral related to the asymptotic behaviour that tends to zero when the ellipse collapses in a crack, was neglected also for a finite value of the notch tip. The accuracy regarding the notch tip methods greatly increases up to the point that the crack could be replaced by a circle in engineering applications [19].

The aim of the paper is to give a simplified procedure for the evaluation of the stress intensity factors of cracks subjected to in-plane mixed mode loadings (mode I plus mode II). The procedure is performed to give an engineering response both for numerical or experimental investigations. The crack is substituted by an

E-mail address: paolo.livieri@unife.it

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equivalent hole and the stress analysis is addressed on the hoop stress on the free border. The procedure is validated by means of strain gauge measurements by directly attaching the strain gauge to the free border of the bore, or to the surface of the plate near the border. The results obtained experimentally are compared with those given by analytical or numerical FE analysis.

2. Hoop stress along the notch border of elliptical notches

The stress along the free border of an elliptical notch was obtained by Inglis in the classical solution reported in Ref. [20]. The ellipse was considered as an isolated notch under remote loading. Recently, by making use of the generalised plane strain hypothesis, an approximate stress field theory has been developed by Zappalorto and Lazzarin [21]. The generalised plane strain hypothesis reduces the notch problem into a common bi-harmonic equation governing the solution of the plane problem and into a harmonic equation governing the antiplane elasticity problem. They considered the case of a slim inclined elliptic hole in a finite thickness plate subjected to a remote tensile load.

When the ellipse cannot be considered an isolated notch or a stress assumes a particular configuration as in Fig. 1, for a linear elastic material, the hoop stress along the free border of the ellipse can be evaluated with high accuracy by considering the solution proposed in Ref. [19].

With reference to Fig. 1, the σ_θ hoop stress on the free border on an ellipse with semi-axis (a, b) is given by:

$$\sigma_\theta = \lambda_1\sigma_{\theta,1} + \lambda_2\sigma_{\theta,2} + \lambda_3\sigma_{\theta,3} \tag{1}$$

where the λ_i are linear coefficients of combination that establish the final shape of the stress along the free border, and the dimensionless $\sigma_{\theta,i}$ stress functions result:

$$\sigma_{\theta,1} = \frac{e^{2\xi_0}}{1+2\frac{a}{b}}\left[\frac{\sinh 2\xi_0(1+e^{-2\xi_0})}{\cosh 2\xi_0 - \cos 2\eta} - 1\right] \tag{2}$$

$$\sigma_{\theta,2} = \frac{2abe^{2\xi_0}}{(a+b)^2}\left[\frac{\sin 2\eta}{\cosh 2\xi_0 - \cos 2\eta}\right] \tag{3}$$

$$\sigma_{\theta,3} = \left[\frac{e^{2\xi_0}\cos 2\eta - \sinh 2\xi_0 - 1}{\cosh 2\xi_0 - \cos 2\eta}\right] \tag{4}$$

where $\xi_0 = \operatorname{arcth}\frac{b}{a}$, $x = a\cdot\cos\eta$ and $y = b\cdot\sin\eta$ with $\eta \in [0, 2\pi]$.

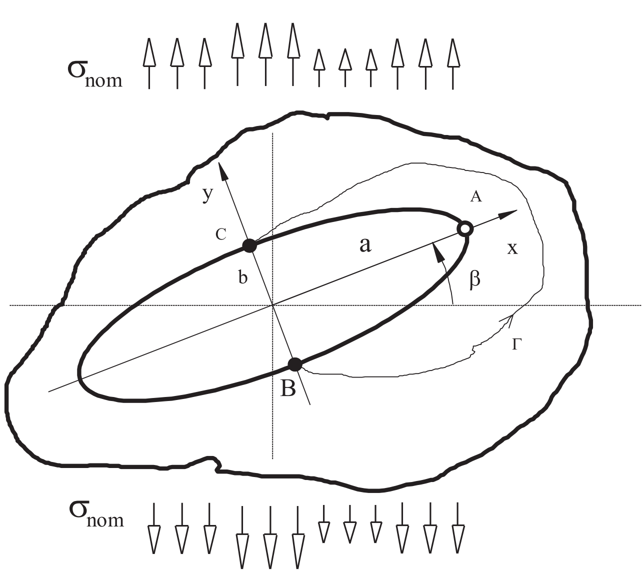

Fig. 1. Ellipse in a body under remote loading.

The stresses $\sigma_{\theta,1}$ and $\sigma_{\theta,3}$ are symmetrical whereas $\sigma_{\theta,2}$ is a skew-symmetric function. The accuracy in stress approximation was discussed in [19] and the stress can be acceptably estimated with an average percent error ranging between 1% and 3%. The percent error $e\%$ is defined in an interval $[\alpha_1, \alpha_2]$ with respect to the stress $\sigma_{\theta,\mathrm{FE}}$ evaluated by means of finite element (FE) analysis:

$$e\% = \int_{\alpha_1}^{\alpha_2} |\sigma_{\theta,\mathrm{Eq.1}} - \sigma_{\theta,\mathrm{FE}}|d\eta \cdot 100 \Big/ \int_{\alpha_1}^{\alpha_2} |\sigma_{\theta,\mathrm{FE}}|d\eta \tag{5}$$

In this work, we consider only the case of circular notches because a circular hole is easier to obtain than an elliptical one from an experimental point of view. So that, the dimensionless stress $\sigma_{\theta,i}$ (2)–(4) can be simplified as follows:

$$\sigma_{\theta,1} = \frac{1}{3} + \frac{2}{3}\cos 2\eta \tag{6}$$

$$\sigma_{\theta,2} = \sin 2\eta \tag{7}$$

$$\sigma_{\theta,3} = 2\cos 2\eta - 1 \tag{8}$$

Fig. 2 shows an example of stress approximation by means of stresses (6)–(8). The λ_i linear coefficients are calculated by imposing a minimum averaged scatter between Eq. (1) and FE results on interval $[0, \pi]$. The coefficients $\lambda_i/\sigma_{\mathrm{nom}}$ are reported in Fig. 2. Obviously, other choices to evaluate the coefficients λ_i are possible, for example, by imposing the exact agreement in three points between σ_θ and $\sigma_{\theta,\mathrm{FE}}$. In the case of Fig. 2, the $e\%$ is 3.9% in the interval $[0, \pi]$.

In order to check Eqs. (6)–(8), an experimental investigation could be made by means of resistance strain gauges as will be shown in a next section. If the thickness of the plate is sufficiently large, the strain gauge can be positioned on the free border with the axis exactly on the middle plane. Then, the trend of the hoop stress σ_θ can be calculated directly through the measured strain by imposing a plane strain or plane stress condition. On the other hand, if the thickness is only a few millimetres or the hole is not large enough with respect to the grid of the strain gauge a direct measurement on the free border of the hoop strain is not possible. So that an alternative way should be used. Accurate FE analysis has shown that the first invariant stress tensor I_θ has a hyperbolic trend near the hole border also far from the former case proposed by Kirsh [22]. On the basis of this observation, we assume that the trend of I_θ will be hyperbolic in the form:

$$I_\theta = A + \frac{B}{r^2} \tag{9}$$

A and B being two parameters that depend on the boundary condition. If one knows the value of I_θ at two different locations, A and B can be easily evaluated and then also the stress at the free border because σ_θ agrees with I_θ under plane stress conditions.

Fig. 3 shows a comparison between the hoop stress calculated directly on the free border and that has been extrapolated with Eq. (9). The agreement is very satisfactory. Furthermore, from Fig. 3, the trend of I_θ as a function of η angle can be expressed as the sum of dimensionless stress function Eqs. (6)–(8) previous assessments of λ_i coefficients.

3. Stress intensity factor assessments

Keeping in mind the stress intensity factors of cracks in flat plates, the crack can be replaced with an equivalent ellipse. If we relate the maximum hoop stress to the SIF as in Refs. [10,14], the tip notch radius has to be very small compared with the crack length. The new approach based on the J-integral could be used to obtain high accuracy with a tip notch radius that is not small enough [19]. As a latter simplification, the ellipse can be replaced by a circular hole as reported in Fig. 4. The high accuracy obtained