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A coupled finite element and element-free Galerkin approach for the simulation of stable crack growth in ductile materials

Sachin Kumar, I.V. Singh *, B.K. Mishra

Department of Mechanical and Industrial Engineering, Indian Institute of Technology, Roorkee 247 667, Uttarakhand, India

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ABSTRACT

In the present work, a coupled finite element (FE) and element free Galerkin (EFG) approach has been proposed for the simulation of two-dimensional stable crack growth problems. In the coupled approach, EFGM has been used in a region near to the crack whereas FEM is utilized in the rest of the region to exploit the advantages of both the methods. In the coupled approach, a ramp function has been used in the transition region to obtain the resultant shape functions. Three problems i.e. crack growth in compact tension specimen, crack growth in triple point bend specimen and crack growth in bi-metallic triple point bend specimen are solved using *J*–*R* curve under plane stress condition to demonstrate the effectiveness of the proposed approach in crack growth problems. These simulations show that the results obtained by coupled approach are in good agreement with the literature results.

1. Introduction

In ductile materials, a significant amount of crack growth occurs prior to failure. Therefore, numerical simulation of the stable crack growth in ductile materials is needed to ensure the maximum utility of the material. The theory of linear elastic fracture mechanics (LEFM) has been widely used by many researchers to investigate structural component reliability and life expectancy. However, the validity of LEFM is limited to brittle materials. The concept of LEFM does not remain valid for ductile materials undergoing large plastic deformations. A typical ductile fracture involves the following processes: (i) crack initiation; (ii) stable crack growth; and (iii) instability. The theory of elastic–plastic fracture mechanics is used to characterize the plastic behavior of a material. Many criteria have been proposed in the past to characterize failure of ductile materials, for example, the CTOD [1,2], the *J*-integral [3,4], the tearing modulus [5] and strain energy approach [6]. Morrison and Karisallen [7] presented a comparison of CTOD and *J* formulation for triple point bend specimens, and found a good agreement between the two criterions. Amstutz et al. [8] proposed that the CTOD/CTOA values remains constant except at the initial stage of crack growth for thin 2024-T3 aluminum specimens. Newman et al. [1] investigated that the crack tip opening angle (CTOA) remains constant behind the crack tip during the stable crack growth. Lam et al. [2] found that the initial values of CTOD/CTOA are high due to the transition from crack blunting to stable crack growth, and remains constant for further stable crack growth.

Over the years, several numerical methods have been developed to simulate the problems of fracture mechanics such as the boundary element method [9], the finite element method [10], meshfree methods [11] and the extended finite element method [12,13]. The modeling of crack propagation is very difficult in standard finite element due to the necessity of conformal mesh. XFEM is one of the most widely used numerical techniques to solve crack propagation problems. In XFEM, conformal meshing is not required, but element distortion issue cannot be avoided in case of large deformation problems. Keeping these issues in mind, meshfree methods would be the ideal choice for crack growth modeling involving large deformation since these methods require only nodal data for the domain discretization.

In last two decades, a class of meshfree methods such as the smooth particle hydrodynamics [14], the diffuse element method [15], the element free Galerkin method [11], the meshfree local Petrov–Galerkin method [16] and the reproducing kernel particle method [17] has been developed to simulate the moving domain problems. The fundamental of all meshfree methods is the requirement of a scattered set of nodal points for the domain discretization. Among all these meshfree methods, element free Galerkin method (EFGM) has been widely used for fracture mechanics problems due to its simplicity. In 1994, Belytschko and his coworkers [18] used EFGM for the modeling of static crack growth problems. In EFGM, the shape functions are constructed by the moving least square approximation scheme [19]. Since, these shape functions do

* Corresponding author. Tel.: +91 1332 285888; fax: +91 1332 285665.
E-mail address: ivsingh@gmail.com (I.V. Singh).

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Nomenclature

$\mathbf{a}_J$	degrees of freedom associated with Heaviside functions
$\mathbf{a}(\mathbf{x})$	vector of unknown coefficients
$\mathbf{b}$	body force per unit volume
$\mathbf{b}_K^{\alpha}$	degrees of freedom associated with asymptotic functions
$\mathbf{B}$	matrix of shape functions gradients
d_{max}	scaling parameter
d_{mI}	domain of influence
$\mathbf{D}^{ep}$	elasto-plastic constitutive matrix
E_{ij}	Green–Lagrange strain
f	yield function
$\tilde{G}$	shear modulus
$\mathbf{G}$	matrix of derivatives of shape functions
$H(\mathbf{x})$	Heaviside function
$\mathbf{I}$	identity matrix
J_{cr}	critical J-integral
k	hardening parameter
$\mathbf{K}^{\mathrm{mat}}$	material tangent stiffness matrix
$\mathbf{K}^{\mathrm{geo}}$	geometric stiffness matrix
$\mathbf{K}_T$	total tangent stiffness matrix
m	order of basis function
$\mathbf{M}_{\boldsymbol{\sigma}}$	matrix of Cauchy stress components
$\bar{n}$	hardening exponent in Ramberg–Osgood model
$\mathbf{N}$	matrix of standard finite elements shape functions
$\overline{\mathbf{N}}$	matrix of transition elements shape functions
$\mathbf{p}^T(\mathbf{x})$	vector of basis functions
Q	plastic potential function
r	radial distance from the crack tip
$R(\mathbf{x})$	ramp function
S_{ij}	second Piolo–Kirchhoff stress
$\mathbf{u}$	displacement vector
$\mathbf{u}_I$	nodal parameter associated with node I at $\boldsymbol{x}_I$
$w(r)$	weight function
W	strain energy density
ε_{ij}	strain tensor
ε_{ij}^e	elastic strain tensor
ε_{ij}^p	plastic strain tensor
$\boldsymbol{\sigma}$	Cauchy stress tensor
σ_y^0	initial yield stress
σ_{ij}'	deviatoric stress tensor
σ_{kk}	hydrostatic stress
$\tilde{\lambda}$	Lagrange multiplier
$d\lambda$	plastic multiplier
ν	Poisson's ratio
δ_{ij}	Kronecker delta
θ	polar angle with respect to crack tip
$\bar{\alpha}$	material parameter in Ramberg–Osgood model
$\beta_{\alpha}(\mathbf{x})$	crack tip asymptotic functions
$\chi(\mathbf{x})$	level set function
Ω	total domain
Ω^{FE}	finite element sub-domain
Ω^{EFG}	element free Galerkin sub-domain
Ω^{TE}	transition sub-domain
$\boldsymbol{\Phi}$	matrix of element free Galerkin shape functions
$\bar{\boldsymbol{\Phi}}$	matrix of shape functions
Γ_u	prescribed displacement boundary

Abbreviations

CMOD	crack mouth opening displacement
CT	compact tension
CTOA	crack tip opening angle
CTOD	crack tip opening displacement
EFGM	element free Galerkin method
EPFM	elasto-plastic fracture mechanics
FEM	finite element method
LEFM	linear elastic fracture mechanics
MLS	moving least square
PU	partition of unity
TPB	triple point bend
XFEM	extended finite element method

not satisfy the Kronecker delta property, hence the essential boundary conditions cannot be imposed directly. To overcome this problem, several techniques have been proposed such as Lagrange multiplier method [11], modified variational principle [20] and coupling of FE–EFG [21]. In 1997, Mukherjee and Mukherjee [22] proposed a technique, based on discrete norm, to palliate the problem of imposition of essential boundary conditions in EFGM.

Till date, the most of the developments in the EFGM were focused mainly in the areas of fracture mechanics [23,24], vibration [25], metal forming [26] and heat transfer [27]. In 2001, Li and Belytschko [28] used the total Lagrangian EFGM formulation for the analysis of contact problems in metal forming. Xin et al. [29] investigated the effectiveness of the EFGM by comparing the results of extrusion process with those obtained using commercial software. Later on, a lot of research work has been carried out using EFGM [30,31] to analyze the three-dimensional linear-elastic fracture mechanics problems but the EFGM has got inferior computational efficiency as compared to FEM and XFEM. Although, the some burden of computational cost is being alleviated with the advancement in computer technologies but it still remains a challenging task for large scale problems. Few approaches have been developed to overcome this problem. One approach is the coupling of EFGM with standard FEM. In this approach, the EFGM has been implemented in the part of the domain while the rest of the domain is modeled with FEM.

Many investigators used different techniques to couple meshfree (EFGM) with FEM. Krongauz and Belytschko [21] used a string of finite elements along the essential boundaries. The combined shape functions are constructed for the transition region, and essential boundary conditions are applied directly at the finite element nodes. Hegen [32] employed Lagrange multipliers to connect the meshfree and finite element regions. Karutz et al. [33] presented the concept for an adaptive coupling of the finite element and element free Galerkin meshing. Although, few studies were performed on elastic problems using coupled FE–EFG approach but the study of elasto-plastic crack growth problems with large deformation has not been performed so far. Therefore, in the present work, an adaptive coupled FE–EFG approach has been used to simulate the nonlinear behavior of materials. Geometric nonlinearity due to large deformation is modeled using the updated Lagrangian approach. Standard Newton–Raphson technique is used for the solution of nonlinear equations. To improve the accuracy and reduce the computational time, EFGM has been used only in a region near the crack, while FEM has been used away from the crack. A ramp function is employed in the transition regions to obtain the resultant shape functions, which comprises both FE and EFG shape functions. The ramp function varies linearly from FE boundary to EFG boundary. Elastic-predictor and plastic-corrector algorithm [34] has been employed for stress computation. Von-Mises yield criterion [35] with isotropic hardening has been

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