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Letter

Thermoelastic vibrations in a thin elliptic annulus plate with elastic supports

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HIGHLIGHTS

- The axisymmetric temperature distribution is determined by the heat conduction differential equation and its corresponding boundary conditions by employing a unified integral transform technique by use of Mathieu functions and modified Mathieu functions.
- The solution of thermally induced vibration of the plate with both ends encased with elastic supports is obtained by employing an integral transform for double Laplace differential equation.
- The thermal moment is derived on the basis of temperature distribution, and its stresses are derived based on resultant bending moments per unit width.
- The numerical calculations of the distributions of the transient temperature and its associated stress distributions are shown in the figures.

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ABSTRACT

In this study, integral operational methods are used to investigate the thermally induced transverse vibration of a thin elliptic annulus plate with elastic supports at both radial boundaries. The axisymmetric temperature distribution is determined by the heat conduction differential equation and its corresponding boundary conditions by employing a unified integral transform technique by use of Mathieu functions and modified Mathieu functions. The solution of thermally induced vibration of the plate with both ends encased with elastic supports is obtained by employing an integral transform for double Laplace differential equation. The thermal moment is derived on the basis of temperature distribution, and its stresses are obtained based on resultant bending moments per unit width. The numerical calculations of the distributions of the transient temperature and its associated stress distributions are shown in the figures.

The thermoelastic vibration analysis is necessitated in many engineerings science applications such as design and computation of structures and mechanical devices, maintenance or predictions of breakdown. These reasons make the study of thermoelastic vibrations in structures and the control of their behaviour under dynamic loads an interesting study. There are a significant number of circumstances in which it is possible to diminish, but not to remove the dynamic forces that excite a system provoking a vibratory behaviour on it. Even loads produced by an earthquake or by wind are unpredictable variables that subject structures to variable dynamic loads. Hence, the displacements caused by dynamic loads, either in the form of thermal or pressure, need to be reduced. This thermoelastic analysis becomes relevant not only for the prediction of thermal displacements, and the related deformations and tensions, but also for their control. Therefore, some theoretical studies concerning the problems of various elastic structural vibrations using different methods have been testified so far. Compared to the cases of rectangular and circular plates, the transverse vibration characteristics of elliptical plates has far less been studied, due to the complexity of obtaining an analytic solution as well as the inadequacy of computational tools for finding numerical solutions. In this paper, we propose a theoretical problem of

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thermoelastic vibrations in a thin elliptic annulus plate with elastic supports that are used in a variety of engineering applications due to its suitable adaptability with experimental results.

A short history of the research investigations associated with the thermoelastic vibrations insights using various operational and approximate methods like the Ritz energy method, Galerkin's method, finite element models and perturbation theory are reviewed that is used to solve the system. Several authors have studied the flexural vibration problems in elliptic structures since the pioneering works of Mathieu on the vibration of elliptic membranes in the nineteenth century. For example, Chen et al. [1] introduced three displacement functions to decompose three displacement components so that the three-dimensional equations of motion of a transversely isotropic body are uncoupled. Further expanding these functions in terms of orthogonal series, the equations of free vibration problem of a transversely isotropic cylindrical shell with ends simply supported are simplified. Sato [2, 3] obtained solutions for composite elliptical membrane consisting of confocal elliptical parts under certain conditions during free as well as forced vibration analysis. Hutchinson and El-Azhari [4] use exact series solutions to the exact governing equations and present highly accurate resonance frequencies of stress-free cylinders by the linear three-dimensional theory of elasticity. Yii and Buchanan [5] use the finite element method to study the effect of Poisson's ratio on the resonance frequencies of free finite hollow cylinders. El-Raheb [6] analyses inhomogeneous hollow cylinders with normal components of time-dependent displacement and stress specified over different parts of the same surface. Gaunaurd and Everstine [7] analyse a hollow elastic cylinder encased in a tube and subject to uniform static compression at the ends. Hasheminejad and Mirzaei [8] analyse a simply supported elastic circular cylinder of finite length with an eccentrically located inner circular cavity, zero stress on the curved surfaces, and shear diaphragm end conditions. Haider [9] investigated the nonlinear forced vibrations of a thermally loaded annular plate with clamped-clamped immovable boundary conditions in the presence of a three-to-one internal resonance between the first and second axisymmetric modes. Kukla [10] investigated the thermally induced vibration of a homogeneous thin circular plate and obtained the solution by using the Green's function method. Recently an excellent article by Yao et al. [11] provides a detailed analysis of free vibration of the clamped elliptical plate, when temperature and stress fields are coupled, with the utilisation of Galerkin's method. Youssef [12] investigated the generalised thermoelastic vibration of a bounded nano-beam resonator induced by ramp type of heating using Laplace transforms. Bera [13] developed an approximate theory which can seem quite appropriate for handling issues of plates, significantly in elliptical coordinates system, both for simple and arbitrary shapes. The solution of problems with bending of a plate was adopted by the rough arrange of the outline of the deflected surface of the plate being physically compatible with the kind of fastening at the boundary, the character of the surface loads, and also the geometrical form of the plate. Hasheminejad et al. [14] formulated eigenfunction expansion in terms of transcendental Mathieu and modified Mathieu functions employed to present the first known exact time-domain series solution for transverse vibrations of a uniform elastic membrane of elliptical shape under arbitrary loading and initial conditions. Ebenezer and Ravichandran [15] analysed free vibrations of cylinders with free or fixed boundaries and forced vibrations of cylinders with specified non-uniform displacement or stress on the boundaries using three series solutions. Kukla [16] derived the Green's function of the Helmholtz operator in an elliptical region and obtained a solution in the form of a double series of Mathieu functions. Very recently, Bhad et al. [17, 18] and Dhakate [19] determined the thermally induced transverse vibration of a uniform thin elliptical object using few new integral transform methods. Though, it has been proved that ample cases of thermoelastic vibration in solids have led to various technical problems in mechanical applications in which heat produced is rapidly sought to be transferred or dissipated.In fact, the thermally induced strain due to cyclic changes in temperature within the flexible range of plate produces a response to transverse vibration in objects. From the above reviews of previous literature indicates that there exist some notable analytical procedures that utilise various analytic, semi-analytic, or approximate numerical methods to investigate the vibrational characteristics. The thermally induced vibration problem in elliptical profile with various complicating effects has received no rigorous analytical or numerical solutions describing the thermoelastic response of solid or annular elliptical structures. The primary purpose of the current work is to take advantage of unified integral transform technique by use of Mathieu functions and modified Mathieu functions to fill this significant gap to obtain the exact solution. The presented analytical model is of both academic and technical interest primarily due to its inherent value as a canonical problem in structural thermodynamics and can serve as the benchmark for the evaluation of other solutions obtained by approximate computational techniques or asymptotic approaches. It can form an invaluable guide for design engineers in assessing the effects of changing the plate eccentricity and edge conditions for elliptical structural components in various physical and technological applications.

This study aims to determine the thermal bending stresses using the exact formal solution of the partial differential equation prevailing the transverse vibration in terms of Mathieu functions and modified Mathieu functions. During problem formulation, we have initially solved heat conduction equation using the integral operation method. Secondly, we shall study the thermoelastic symmetrical vibration with elastic supported boundaries conditions using the theory of integral transform. Finally, the analytical solution for the thermal stress components is obtained based on resultant bending moments per unit width. Results of four other cases corresponding to the constant temperature distribution and temperature prescribed to a point on the upper face of the disk have been briefly described. The results presented here will be more useful in engineering problems particularly in vibrational analysis and in determining the state of strain in elliptical disk constituting foundations for reactors, pressure vessels, furnaces, etc.

It is assumed that a thin elliptical annulus plate is occupying the space $D:\left\{(\xi,\eta,z)\in R^3:\xi_i<\xi<\xi_o, 0<\eta<2\pi, 0<z<\ell\right\}$ under transient temperature state having internal heat source within it, while supports at the boundaries as non-rigid. The geometry of the elliptical annulus plate indicates that an elliptic-cylindrical coordinate system (ξ,η,z) is the most appropriate choices of the reference frame, which are related to the rectilin-