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# Natural frequency optimization of 3D printed variable-density honeycomb structure via a homogenization-based approach

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ABSTRACT

It is well-known that the effective mechanical properties of cellular structures can be tuned by varying its relative density. With the advancement of 3D printing, variable-density cellular structures can be fabricated with high precision using this emerging manufacturing technology. Taking advantage of this unique ability to fabricate variable-density cellular structure, an efficient homogenization-based topology optimization method for natural frequency optimization is presented in this work. The method is demonstrated using a cantilevered plate with a honeycomb structure and is validated by detailed finite element analysis and experiment. It is shown that the optimal design can be fabricated by 3D printing and shows significant enhancement in natural frequency and reduction in weight.



## 1. Introduction

Natural materials (e.g. wood and bone) have drawn much attention for many years and it has been found that cellular structures in those biological materials reduce the density of materials but maintain remarkable mechanical performance [1,2]. Inspired by these natural materials, cellular structures such as foams, honeycombs, lattice structures, and other similar structures have been engineered for their high performance in energy absorption and thermal insulation in combination with their relative low weight [3–7]. Traditionally, engineered cellular structures are manufactured using methods such as vapor deposition, foaming, casting, and sintering [3,8]. Although these methods still play a dominant role in producing cellular structures, they have manufacturing difficulties when dealing with complex cell geometry and spatially-varying porosity. However, additive manufacturing (AM) technology can overcome these difficulties since it has the ability to fabricate parts with almost unlimited geometric complexity [9–12].

In fact, AM had already been employed to fabricate uniform-density or stochastic cellular structures [13,14]. In addition, cellular structures with variable density can also be fabricated easily to maximize the potential of this technology. First, it allows the mechanical properties to be varied in different regions in order to tailor the structure for specific design requirements. Second, it leads to savings in material and energy consumption, which is much more important for AM than for traditional subtractive manufacturing, because geometric complexity is almost free with AM. Third, according to previous simulation results, cellular structures can be designed with increased resistance against defect propagation [15–18]. For these reasons, variable-density cellular structure is a promising candidate for AM application.

In order to take advantage of the ability of AM to fabricate variable-density cellular structures, a design optimization method that is able to optimize their designs is necessary. One such method is to optimize the strut size or wall thickness after obtaining the designed cell structure [19–21]. By using this method, both the structural and cell levels can be optimized, but the drawback is that it is only applicable to some regular strut based cellular structures. Another efficient design optimization method is already proposed in our previous work for optimizing minimum compliance design of AM cellular structure [22,23]. In this homogenization-based method [24,25], the elastic properties of a given periodic cellular structure are first obtained through computational homogenization as a function of its relative density; then the homogenized model is employed in a standard topology optimization algorithm

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to obtain the optimal relative density distribution, which is finally converted to the actual variable-density cellular structure. The advantage of this homogenization-based optimization method is that it is easy to understand and implement; however, it can only predict the optimal topology at the structural level instead of within each cell.

Considering its efficiency, the homogenization-based method will be extended to achieve natural frequency optimization of AM cellular structures with variable densities in this paper. Natural frequency optimization plays an important role in the design of many engineering structures that are subjected to high-amplitude and frequent vibrations [26]. These structures range from wind turbine blades and engines to structures in seismically active regions, one of their major concerns is dynamical failure caused by vibrations at their natural frequency. Therefore, AM cellular structures designed via topology optimization have a great opportunity of making an impact in vibration and control engineering.

Since the design optimization of variable-density cellular structures for static problem (stress and compliance) has been successfully applied [27,28], the key issues we aim to resolve now is whether the homogenized model for cellular structure employed in the proposed design optimization method is valid for dynamical design problems. To demonstrate its validity, both detailed finite element analysis and dynamical experiments on the optimized structure will be carried out, and the results will be compared with solution obtained from the homogenized model. The optimal design cellular structure can be easily realized by using AM in order to conduct the experiment.

This paper aims to establish an efficient computational method for optimizing the internal topology of variable-density cellular structures to maximize the fundamental frequency. The paper is organized as follows: Section 2 gives a brief introduction of the whole design method while Sec. 3 presents a method that could solve optimization problems with simple or multiple frequencies [29,30]. In Section 4, we present the homogenization of the honeycomb structure in order to obtain a relationship between its effective mechanical relationship and the relative density. Section 5 discusses how reconstruction of the honeycomb structure is performed once the optimized relative density distribution is obtained. In Section 6, design method was performed on a thin plate. The performance of the whole design method is compared with detailed finite element analysis and modal testing in Section 7 and 8. Conclusion will be given in Section 9.

## 2. Overview of design method

As illustrated in Fig. 1, the proposed cellular structure design method consists of three steps: (i) homogenization, (ii) optimization and (iii) reconstruction. To demonstrate the proposed method, we choose to optimize the design of a honeycomb structure in a simple cantilever plate. In the homogenization step, scaling laws (which represent the relations between the elastic properties of the cellular structure and its relative density) are obtained through conventional computational homogenization [23]. During homogenization (Fig. 1(a)), the relative density of the cellular structure is varied by changing the hole size of honeycomb, and finite element analysis (FEA) is carried out to determine the elastic constants for each relative density. The scaling laws obtained are then directly employed in a standard topology optimization algorithm to obtain the optimal relative density distribution for the frequency optimization problem at hand (Fig. 1(b)). The actual cellular structure is reconstructed from the optimized relative density distribution through using a relationship between the relative density and hole size of the honeycomb (Fig. 1(c)). More details for each of these steps will be elaborated in the following sections.

## 3. Optimization method for eigenfrequency problem

The eigenfrequency maximization problem in the finite element (FE) framework takes the following mathematical form:

$$\begin{cases} \max\ \omega_j = \sqrt{\dfrac{u_j^T K u_j}{u_j^T M u_j}} \\ st. \begin{cases} \sum \rho_e V_e - V^* \le 0 \\ 0 \le \rho_{\min} \le \rho_e \le \rho_{\max} \le 1 \end{cases} \end{cases} \tag{1}$$

with the first equation arising from the dynamic eigenvalue problem:

$$K u_j = \omega_j^2 M u_j, j = 1, \ldots, n \tag{2}$$

where $K$ and $M$ are the stiffness and mass matrices of the structure, respectively, while $\omega_j$ and $u_j$ are the $j$th eigenfrequency and corresponding eigenvector. Meanwhile, $\rho_e$ and $V_e$ in Eq. (1) represent the relative density and volume of element $e$, respectively. The first constraint in the equation means the volume fraction must be no less than the volume constraint $V^*$. The second constraint indicates that the relative density is bounded by the minimum and maximum relative densities, $\rho_{\min}$ and $\rho_{max}$.

The optimization problem in Eq. (1) can be solved by a number of different methods such as the MMA method, which is based on a special type of convex approximation where sensitivity analysis becomes a key part of deriving the specific optimization algorithm. Usually, sensitivity could be obtained by taking derivative directly, but sensitivity analysis may fail to converge for vibration problems involving structures having geometric symmetries [31,32]. This is because there may be repeated eigenvalues (i.e. distinct modes at the same natural frequency) in the problem. For repeated eigenvalue problems, assume that one of the natural frequencies $\omega$ has multiplicity N:

$$\omega = \omega_j, j = n \cdots n + N \tag{3}$$

Unlike simple eigenvalue problems, sensitivity analyses for repeated eigenvalues cannot be obtained directly as they are non-differentiable with respect to the design variable in the common mathematical sense [31,32]. According to Seyranian [33,34], this difficulty can be overcome by utilizing perturbation analysis in the sensitivity analysis [35], which leads to the following natural frequency maximization problem written in terms of the change in relative density and frequency, $\Delta\rho_e$ and $\Delta{\omega_j}^2$, respectively [35]:

$$\begin{cases} \max\limits_{\beta, \Delta\rho_e} \beta \\ st. \begin{cases} \beta - \left[\omega_j^2 + \Delta\omega_j^2\right] \le 0 \\ \det\left[f_{sk}^T \Delta\rho_e - \delta_{sk}\Delta\omega_j^2\right] = 0 \\ \sum(\rho_e + \Delta\rho_e) V_e - V^* \le 0 \\ 0 \le \rho_{\min} \le \rho_e + \Delta\rho_e \le \rho_{\max} \le 1 \end{cases} \end{cases} \tag{4}$$

where $\beta$ is the new bound variable and $\delta_{sk}$ is the Kronecker delta, while $f_{sk}$ are generalized gradients for the sensitivity analysis of the optimization problem in Eq. (4):

$$f_{sk} \equiv \Phi^T \left(K'_{\rho_e} - \omega_j^2 M'_{\rho_e}\right) \Phi, (s, k = n, \cdots, n + N - 1) \tag{5}$$

where $\Phi$ consists of the eigenvector associated with the N-fold eigenvalue. If only simple case is considered and no repeated frequency shows up, $\Phi$ would be the corresponding eigenvector. In this case, the gradient obtained using Eq. (5) will be identical to that in traditional MMA method. $K'_{\rho e}$ and $M'_{\rho e}$ denote the partial derivatives of the stiffness and mass matrices with respect to the design parameter $\rho_e$. Compared with the standard problem in Eq. (1) in which the only design parameter is $\rho_e$, the new unknowns

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