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Non-local failure theory and two-parameter tensile strength model for semi-circular bending tests of granitic rocks

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ABSTRACT

Several indirect tension tests have been proposed and widely used to quantify rock tensile strength due to difficulties associated with experimentation in direct tension tests. However, discrepancies exist between rock tensile strengths obtained from the direct tension tests and those from indirect tension tests. The non-local theory has been used to reconcile such differences. However, the physical meaning of the characteristic material length δ in the non-local theory is still unclear. We investigate the physical meaning of the characteristic length δ by testing three typical granitic rocks featuring different grain sizes. An indirect tension method - semi-circular bending (SCB) test is applied to obtain the flexural tensile strength for these rocks. The specimens are perfectly split into two quadrants by a main crack through the center of the specimen. The flexural tensile strengths of three rocks generally decrease with the span distance. From flexural tensile strengths under different testing conditions, the optimized values of the characteristic material length δ and the rock intrinsic tensile strength are determined for these three rocks. It is found the tensile strengths of three rocks predicted by the non-local theory match well with those obtained from independent experiments using the Brazilian disc method. The non-local failure theory is used to explain the observation that the flexural tensile strength of three rocks decreases with the span distance. Moreover, the material characteristic length δ correlates well with the rock grain size, and the physical meaning of material characteristic length can be considered as the scaled average grain size of rocks. Based on the successful application of the non-local theory, a two-parameter tensile strength model for rocks is proposed.

1. Introduction

Underground opening is common in many important geological and geophysical applications including mining, petroleum, defense infrastructure and hydropower. Tensile stress and tensile stress gradients are often encountered in underground opening although the general in-situ stress state for underground rocks is normally compressive. Due to the much smaller tensile strength of rocks as compared to their compressive strength, tensile failure is the main and significant failure mode of rocks. Consequently, tensile strength is an important material parameter for rocks.

Tensile strength is defined as the failure stress under pure uniaxial tensile loading, the direct tension method (e.g., direct pull test) is thus the most suitable method for the determination of rock tensile strength.[1] However in practice, significant error may be introduced to the direct tension measurement results from stress concentration due to gripping of the rock specimens and misalignment.[2] Hence, several alternative indirect tension tests were proposed to measure the tensile strength of rocks.[3,4] Brazilian disc (BD) test is the most popular indirect tension test for rocks due to ease in alignment and low-cost in experimentation.[3,5,6] Under uniform far-field compression, the bi-axial stress state is induced at the failure spot (i.e., center of the disc) in the BD specimen: the compression along the loading direction and the tensile stress perpendicular to the loading direction. In this case, the tensile stress near the failure spot in the BD rock specimen is uniform.[7] The tensile strength measured using the BD method is thought to be the closest one to the rock intrinsic tensile strength and the small difference between these two has been explained.[2]

However instead of uniform loading condition, bending load is common near underground openings, e.g., flexural tensile stresses are induced at the roof of an underground opening and the failure mode is consequently dominated by the flexural tensile failure. In such cases, stress gradients of varying magnitude and in-situ stresses always accompany the flexural tensile stresses in rocks, depending on both the configuration of the excavation and on the specific rock breaking technique.[8] High tensile stress gradients also exist on a microscale in

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Table 1
The comparison between the flexural tensile strength and tensile strength of rocks.

Rock	Flexural tensile strength (MPa)	Tensile strength (MPa)
Barre granite (BG)	16.5[10]	9.4[5]
Laurentian granite (LG)	26.2[16]	12.8[16]
Bowral Trachyte	7.8[17]	3.7[18]
Gosford Sandstone	11.6[17]	8.6[18]
Carrara Marble	25.1[17]	12.0[18]

the vicinity of flaws, at grain contacts and at pores in rocks. Van de Steen et al.[8] experimentally and theoretically investigated the influence of stress gradients on the flexural tensile failure of rocks. It has been proven that the fracture initiation mainly depends on both the stress gradient and the magnitude of the bending stress. Therefore, both the rock flexural tensile strength and the stress gradient in rocks considerably affect the tensile failure of rocks.

Semi-circular bending (SCB) test is an effective method to study the rock flexural tensile failure in the laboratory. The tensile failure of rock specimen in the SCB method is induced by far-field compression, and the tensile stress at the failure point of the SCB tests is produced by bending load. Along the fracture path in the SCB specimen, there exists a tensile stress gradient of various magnitudes, depending on the configuration of the SCB specimen.[8,9] This usually leads to a quite higher flexural tensile strength of rocks as compared to the tensile strength obtained from the BD tests (as shown in Table 1).[2,3,6,8,10–13] This discrepancy has been reconciled by non-local failure theories.[2,8,10] In these theories, the strength of a body at a point is generally a function of the stress state from a certain representative volume, area or line around that point.[14] It is clear that non-local failure theories always involve the introduction of a characteristic material length δ. Using an appropriate characteristic length value, the intrinsic tensile strength of a material in the non-local theory as obtained from the apparent strength (i.e., flexural tensile strength) values is similar to the tensile strength obtained from the BD tests with a negligible stress gradient.[2,8,10] It has been approved that the characteristic length can be considered as a material parameter, being related to the aggregate or grain size.[15] Nevertheless, the physical meaning of the characteristic material length scale in the non-local theories is still unclear. Hence, the primary objective of this work is to physically determine the meaning of the characteristic material length for rocks.

To achieve the objective of determining the physical meaning of the characteristic material length, we employed the SCB method to obtain the flexural tensile strength for three typical granitic rocks featuring different grain sizes. Based on the different loading conditions, the optimized values of the characteristic material length and the rock intrinsic tensile strength are determined for these three rocks. In addition, the physical meaning of the material characteristic length δ is given by using the rock grain size. Based on the successful application of the non-local theory, a two-parameter tensile strength model for general tensile rock failure is proposed, which provides good and consistent guidelines for rock structural design involving a stress gradient.

This paper is organized as follows. After the introduction, Section 2 reviews the non-local failure criteria. The experimental methodology is introduced in Section 3 and Section 4 describes the rock characteristics. The results along with discussions are presented in Section 5. Section 6 summarizes the entire paper.

2. Non-local failure criteria

There are a number of non-local failure criteria proposed by researchers.[14,19–23] Some of these criteria not only give a good description of experimental data when the stress distribution is close to a uniform stress state, but are also applicable to bodies with cracks and singular stress concentrators. Some generalized forms of non-local failure criteria were presented in the literature,[14,19,24] including a general form of body non-local strength conditions, a general form of point non-local strength conditions, and several functional forms of non-local strength conditions. For non-local failure criteria based on strength evaluation, the generalized form can be expressed as follows:

$$f(s_{ij}(\mathbf{y})) = \sigma_c \tag{1}$$

where f is a function of stress tensor s_{ij}, $\mathbf{y}$ is an analyzed point, σ_c is a material constant that is called the intrinsic tensile strength in this work. For two-dimensional cases, there are three most popular non-local failure criteria based on strength evaluation:[14] fracture criterion based on average stress over a characteristic length or average stress fracture criterion (ASFC),[20,21] fracture criterion based on a minimum stress over a characteristic length or minimum stress fracture criterion (MSFC)[21] and fracture criterion based on a model of fictitious crack with a characteristic length or fictitious crack fracture criterion (FCFC).[22,23] We assume that (ρ, θ) is a local polar coordinate system with the center at the analyzed point $\mathbf{y}$ of a body; $\boldsymbol{\eta}(\theta)$ is a unit vector with angle θ to the coordinate axis; and $\sigma_{\rho\rho}$, $\sigma_{\theta\theta}$ and $\sigma_{\rho\theta}$ are the stress components in this coordinate system.

The ASFC states that the material fails when the tensile strength of material is equal to the average local stress over a specific distance (i.e. a characteristic length) along the prospective fracture direction. It can be written in the following generalized form:

$$\frac{1}{\delta_1}\max_{-\pi<\theta<\pi}\int_0^{\delta_1}\sigma_{\theta\theta}(\mathbf{y}+\rho\boldsymbol{\eta}(\theta))\,d\rho = \sigma_c \tag{2}$$

where σ_c is the intrinsic tensile strength of a body under uniform traction, δ_1 is a characteristic material length. If the maximum direction θ_0 in Eq. (2) is known and the integration is performed along this direction, the ASFC equation can be simplified as follows:

$$\frac{1}{\delta_1}\int_0^{\delta_1}\sigma_{\theta\theta}(\mathbf{y}+\rho\boldsymbol{\eta}(\theta_0))\,d\rho = \sigma_c \tag{3}$$

The MSCF implies that the material fails when the tensile strength of material is equal to the minimum local stress over a characteristic length along the prospective fracture direction. It may be written as:

$$\max_{-\pi\le\theta\le\pi}\left[\min_{0\le\rho\le\delta_2}\sigma_{\theta\theta}(\mathbf{y}+\rho\boldsymbol{\eta}(\theta))\right] = \sigma_c \tag{4}$$

where σ_c and δ_2 are material constants with the similar meaning as those in Eq. (2). With a known maximum direction θ_0 in Eq. (4), MSFC can be rewritten in a simpler form:

$$\min_{0\le\rho\le\delta_2}\sigma_{\theta\theta}(\mathbf{y}+\rho\boldsymbol{\eta}(\theta_0)) = \sigma_c \tag{5}$$

The FCFC indicates that the material fails when the stress intensity factor of material is equal to the critical local stress intensity factor over a characteristic length along the prospective fracture direction. In the FCFC, it is assumed that there exists a fictitious crack with a characteristic length δ_3 originating from the considered point $\mathbf{y}$ of the body. The FCFC may be represented in the following form:

$$\max_{-\pi\le\theta\le\pi}\min_i K_{1i}(\mathbf{y},\theta,\delta_3) = K_{1c} \tag{6}$$

Here K_{1c} and δ_3 are material constants, $K_{11}=K_1(\mathbf{y})$ and $K_{12}=K_1(\mathbf{y}+\delta_3\boldsymbol{\eta}(\theta))$ are the stress intensity factors at the ends of the fictitious crack along the $\boldsymbol{\eta}(\theta)$ direction. If the direction θ_0 of fracture is known, Eq. (6) yields:

$$\min(K_1(\mathbf{y}), K_1(\mathbf{y}+\delta_3\boldsymbol{\eta}(\theta_0))) = K_{1c} \tag{7}$$

For an edge crack beginning from the body boundary, there exists only one stress intensity factor $K_1(\mathbf{y}+\delta_3\boldsymbol{\eta}(\theta_0))$ and the minimum disappears in Eq. (7).

Comparing these three non-local failure criteria, one can see that each of the criterion mentioned above contains two material