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Numerical and analytical study of rate effects on drilling forces under bottomhole pressure

Mohamed Amri*, Gilles Pelfrene, Emad Jahangir, Michel Tijani, Hedi Sellami

MINES ParisTech, PSL-Research University, Geosciences Research Center, 60 bd Saint Michel, Paris, France
VAREL Europe, 2 rue Johannes Kepler, 64000 Pau, France

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ABSTRACT

The scope of this paper is to investigate rate effects on rock cutting process at bottomhole pressure. The solution of the cutting problem is derived in the framework of the theory of poroelastoplasticity. The problem is firstly solved analytically using some assumptions derived from the literature review. Then, a numerical resolution of the fully coupled problem is performed based on the finite element method. These two different approaches show that pore pressure in shear-dilatant rocks decreases as a function of the cutting velocity depending on rock permeability and interstitial fluid properties. Pore pressure change has a hardening effect leading to an increase of the rock drilling resistance.

1. Introduction

The dependence of rock cutting forces to the strain rate has been the subject of several studies in various technical fields. In the field of the Oil & Gas drilling for instance,[1] investigates experimentally and theoretically rate effects on rocks drillability at atmospheric pressure. The author demonstrates that forces acting on individual cutters significantly increase with the cutting velocity. This rate-dependent phenomenon has been explained by the friction process underneath the cutter.[2] explores the potential of the viscoplastic theory in rock cutting process. While the aim of the author is to provide a numerical framework for cuttings fragmentation process, she shows that rate effects due to viscoplasticity are important for typical drilling conditions at atmospheric pressure. Under saturated rocks, as it is the case in most drilling applications,[3,4] have provided a theoretical framework to explain observed rate-effects. They show that the observed hardening effects (drilling forces increase with the cutting speed) are caused by the drop in pore pressure which occurs in shear-dilatant rocks.

Experimental evidence of strain rate effects have also been demonstrated for triaxial tests[5] in low porosity rocks.[6] has developed a numerical model that captures strain rate dependent behaviours like compactive weakening (increase of pore pressure) and dilatant hardening (decrease of pore pressure). This work uses the Discrete Element Method (DEM) to model the mechanical behavior and the Smoothed Particle Method (SPM) to model the fluid behavior and has been applied in the field of offshore mining to analyze how the rock cutting process is affected by fluid pressure in.[7]

Rate effects have also been largely studied in the soil ploughing domain. For instance,[8–10] show theoretically and experimentally that cutting forces increase dramatically with cutting speed in soil ploughing. Authors suggest that these effects are due to the pore pressure drop with the cutting speed which leads to an increase of the effective stress in the soil. The magnitude of rate effects depends on soil hydrodynamic properties, especially permeability and porosity.

A recent series of PDC (Polycrystalline Diamond Compact) single cutter tests have confirmed the existence of similar rate effects in the field of Oil & Gas drilling .[11,12] These studies show that under a bottomhole pressure representative of oil and gas PDC drilling conditions, the cutting speed can strongly impact cutter forces depending on hydromechanical properties of the saturated rock formation. As already suggested in the literature,[4,9] this phenomenon is believed to be related to the transition from the drained to the undrained regime flow conditions as the cutting speed increases.

This phenomenon has already been investigated theoretically. For instance,[4] use an analytical model to quantify the pore pressure change with the cutting speed for low permeability shear-dilatant rocks. Analytical resolution was possible under steady state and partial coupling assumptions. Cutting forces are firstly calculated as a function of pore pressure, which is determined *a posteriori* as a function of cutting speed and dilatancy.[13] develops a fully coupled cutting model based on the theory of elastoplasticity. Hydromechanical coupled equations are solved numerically in the transient regime. The author shows that

* Corresponding author.
E-mail address: mohamed.amri@mines-paristech.org (M. Amri).

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Table 1
Overview of some existing poromechanical cutting models.

Model	$\mathscr{H}\mathscr{M}$ resolution	Coupling	Hydraulics	Mechanics	Dilatancy
[8]	Semi-analytical	Partial	Steady-state	Static	Empirical
[9]	Analytical	Partial	Steady-state	Static	Empirical
[4]	Analytical	Partial	Steady-state	Static	Empirical
[13]	FEM	Full	Transient	Static	Absent (elastic)
[7]	DEM-SPM	Full	Transient	Transient	Elastoplasticity

cutting forces are independent of the cutting speed contrarily to.[4] This can be explained by the fact that in,[13] the rock dilatancy phenomenon has not been modeled in saturated rocks.

[3,4,8,9], dilatancy is a primary cause of rock cutting rate effects in saturated rocks. The purpose of this study is to investigate this phenomenon theoretically as a contribution to previous works (Table 1). In the first part, poromechanical equations applied to the rock cutting process are recalled. In the second part, an analytical solution is derived using partial coupling simplification. This solution aims to improve existing analytical models. Then, a numerical resolution of the fully coupled problem is derived in the framework of poroelastoplasticity theory. Finally, a sensitivity analysis of drilling forces on rate effects is presented.

2. Problem formulation

2.1. Geometric model

Fig. 1 shows a schematic view of the rock cutting process. The PDC cutter, which is supposed to be sharp, makes an angle θ with respect to the vertical axis. The depth of cut and the linear cutting speed are denoted by h and v, respectively. The rock is subjected to the mud pressure p_m and to the pore pressure field p. F_c and F_n represent the tangential force and the normal force, respectively.

Plane strain hypothesis has been used by many authors[4,1,13–20] to model the cutting problem. A recent 3D analysis based on finite element shows that cutting forces for linear cutting and circular cutting is almost identical whenever the depth of cut is lower than the PDC radius.[21] This hypothesis will be used in this paper.

Additional assumptions are considered to model the creation of the rock chip. The analysis is restricted to a finite region Ω near the cutter edge with a boundary $\partial\Omega$ that belongs to the (x, y) plane. Depth of cut h is assumed positive and constant. In the framework of the plane strain hypothesis, the width of the rock chip in the third direction is $l > 0$.

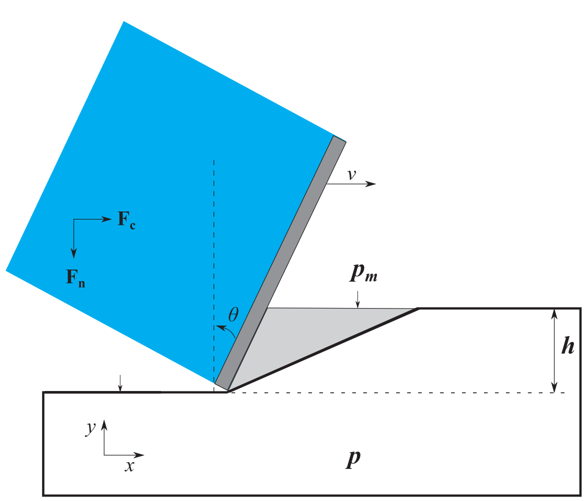

Fig. 1. Geometric model of the rock cutting process.

2.2. Hydraulic model

The region Ω is assumed to be saturated with a low-compressibility fluid (typically oil or water in the liquid form). The general diffusion equation in a fixed Eulerian frame is given by[22]:

$$b\dot{\varepsilon}_v + \frac{1}{M}\dot{p} - \frac{1}{\rho g}\vec{\nabla}.\left(\underline{K}\,\vec{\nabla}p\right) = 0 \tag{1}$$

where b is the Biot coefficient, ε_v is the volumetric strain, p is the pore pressure, M denotes the Biot Modulus, ρ is the fluid density whereas $\underline{K}$ denotes the hydraulic conductivity tensor of second order.

The hydraulic problem is solved in the steady state regime as has been done previously.[4,9] Eq. (1) can be written in a Lagrangian frame (x, y) attached to the cutter. For homogeneous and isotropic materials, this equation can be expressed as a function of the cutting velocity $\vec{v}$ as follows:

$$K\vec{\nabla}.\vec{\nabla}p + \frac{\rho g}{M}\vec{v}.\vec{\nabla}p = -\left(\rho g b\right)\vec{v}.\vec{\nabla}\varepsilon_v \tag{2}$$

Eq. (2) shows that fluid exchange takes place by diffusion ($K\vec{\nabla}.\vec{\nabla}p$) and convection ($\frac{\rho g}{M}\vec{v}.\vec{\nabla}p$) simultaneously. In addition, hydro-mechanical coupling takes place through a source term represented by the right hand side of Eq. (2).

For convenience, the transport Eq. (2) is now written in the following normalized form:

$$\vec{\nabla}.\vec{\nabla}p = -\frac{h}{D}\vec{v}.\vec{\nabla}(p + bM\varepsilon_v) = -2\lambda\vec{i}.\vec{\nabla}(p + bM\varepsilon_v) \tag{3}$$

where $D = \frac{KM}{\rho g}$ is the diffusivity of the fluid through the porous material, $\vec{i}$ is the cutting velocity direction and $\lambda = \frac{h\|\vec{v}\|}{2D}$ is the normalized speed that can also be interpreted as the ratio of $\tau_d = \frac{h^2}{2D}$, the diffusion characteristic time, by $\tau_c = \frac{h}{\|\vec{v}\|}$, the convection characteristic time. Note that λ is equivalent to Péclet number used in heat transfer modeling. Table 2 gives typical values of intrinsic permeability k and λ for different materials encountered in oil and gas drilling assuming a depth of cut $h = 1$ mm and a velocity $v = 1$ m/s when the saturating fluid is water.

2.3. Mechanical model

Let $\underline{\sigma}$ be the Cauchy total stress tensor of second order. According to Biot model,[24] $\underline{\sigma}$ is the sum of two components:

$$\underline{\sigma} = \hat{\underline{\sigma}} - bp\underline{1} \tag{4}$$

where $\hat{\underline{\sigma}}$ denotes the effective stress governing the rock skeleton strains, p is the pore pressure, b is the Biot coefficient and $\underline{1}$ is the unitary tensor of second order.

Existing rock cutting models generally assume that the total stress $\underline{\sigma}$

Table 2
Order of magnitude of the intrinsic permeability k and the dimensionless speed λ for different materials when the saturating fluid is water.

Material	k [m^2][23]	λ
Soils		
Fine sands, silts and loess	$10^{-12} - 10^{-9}$	$10^{-7} - 10^{-4}$
Gravels and sands	$10^{-16} - 10^{-12}$	$10^{-4} - 1$
Rocks		
Limestone	$10^{-16} - 10^{-12}$	$10^{-4} - 1$
Sandstone	$10^{-17} - 10^{-11}$	$10^{-5} - 10$
Clays	$10^{-20} - 10^{-16}$	$1 - 10^{5}$
Granites, gneiss, compact basalts	$10^{-20} - 10^{-16}$	$1 - 10^{5}$
Concrete	$10^{-21} - 10^{-16}$	$1 - 10^{6}$