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Influence of pore pressure on the development of a hydraulic fracture in poroelastic medium

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ABSTRACT

In this paper we demonstrate the influence of the pore pressure on the development of a hydraulically-driven fracture in a poroelastic medium. We present a novel numerical model for propagation of a planar hydraulic fracture and prove its correctness by the analysis of the numerical convergence and by the comparison with known solutions. The advantage of the algorithm is that it does not require distinguishing of the fracture's tips and reconstruction of the numerical mesh according to the fracture propagation. Next, we perform a thorough analysis of the interplay of fluid filtration and redistribution of stresses near the fracture. We demonstrate that the fracture length decreases with the increase of the Biot's number (the parameter that determines the contribution of the pore pressure to the stress) and explain this effect by analysing the near-fracture pore pressure, rock deformation and stresses. We conclude that the correct account for the fluid exchange between the fracture and the rock should be based not only on physical parameters of the rock and fluid, but also on the analysis of stresses near the fracture.

1. Introduction

Mathematical modelling of hydraulically-driven fractures is a highly demanded subject in modern technologies for enhancement of reservoir permeability in hydrocarbon production as well as in geophysical problems related, for instance, to the development of magmatic dykes. Recent progress in the modelling of hydraulic fracture dynamics is described in the review papers[1,2] and citations therein. The early devised but currently widely used models by Khristianovich, Zheltov, Geertsma, and de Klerk (KGD)[3,4] and by Perkins, Kern and Nordgen (PKN)[5,6] assume that the fracture propagates in infinite elastic medium and the fluid exchange between the fracture and the porous reservoir is modelled only as a fracturing fluid loss (leakoff) according to Carter's formula[7] which proposes that the leakoff is inversely proportional to the square root of the exposure time. More advanced models of the leakoff suppose computation of the pore pressure around the fracture by solving the piezoconduction equation[8] although still do not considering the influence of the pore fluid on stresses.

Theoretical study of the impact of the pore pressure on the distribution of stresses near the fracture was carried out in many papers, a detailed review can be found in the Introduction of the dissertation by Y. Yuan.[9] In particular, the additional stiffness of the rock due to the pressure in the vicinity of the fracture was treated as the backstress.[10,11] It was noted that the wellbore fluid pressure needed to open the fracture considerably rises due to the backstress. The same effect leads to the overestimation of the minifrac tests for the in situ minimal principal stress.[12,13] The mentioned facts indicate that the proper account for the action of the pore pressure and the proper modelling of the fluid exchange between the fracture and the porous reservoir is principal for the correct description of the fracture dynamics.

In our paper we extend this observation by performing a thorough analysis of the mutual influence of the pore fluid filtration and the stresses distribution. The analysis is based on the novel numerical model for propagation of a hydraulic fracture in a poroelastic medium, based on Biot's equations.[20] The brief outline of the model is given in our conference paper.[14] Here we provide the complete description and verification of the model, and do the numerical simulations. The numerical solution of the problem is carried out by the finite element method with the use of a modification of the algorithm suggested in.[15] We use an approach of modelling free of explicit tracking of the fracture's tip similar to the one used in.[16] The advantage of our model is that we do not need to rebuild the computational mesh according to propagation of the fracture, which is typical for problems of this type. Also it allows to incorporate all “fracture propagation — fluid flow” coupling in a single weak formulation, which is ready to be solved numerically using standard methods. The rock failure criteria is

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modelled using the cohesive zone model initially proposed by Barenblatt[17] and Dugdale.[18] This model allows us to eliminate the stress singularity at the fracture's tip as well as to integrate the computation of the failure criteria into the numerical algorithm. The correctness of the model is checked by the analysis of the numerical convergence of the algorithm and by comparison with analytical and numerical solutions presented in.[19] In all observed cases we have a satisfactory coincidence of the solutions.

The constructed model is used for the analysis of the influence of the pore pressure on the fracture dynamics. We demonstrate that the dynamics is governed by the two factors: the rate of the medium displacement that modifies the filtration, and by the backstress that significantly increases the pressure inside the fracture. For the relatively high rock permeability these two factors notably increase the leakoff and hence, decrease the length of the fracture. The demonstrated effect is dumped by high reservoir storage coefficient or low rock permeability.

2. Mathematical formulation of the problem

Let us consider a vertical planar fracture of fixed height H, propagating along the straight line denoted as x-axis (Fig. 1a). We direct z-axis upwards and y-axis perpendicular to the plane of the fracture propagation. We suppose that fracture's aperture is constant along the vertical coordinate z, so the plane strain approximation is applicable. This implies, that we can limit ourselves to observing only the central cross-section $z = 0$ of the fracture.

2.1. Equations for the poroelastic reservoir

The poroelastic medium is characterized by its porosity ϕ and permeability $k_r(\mathbf{x})$, with the solid phase displacement $\mathbf{u}(t, \mathbf{x})$, and the pore pressure $p(t, \mathbf{x})$. Pores are saturated with a single-phase Newtonian fluid with the effective viscosity η_r. We make use of the linear Darcy law for the fluid velocity $\mathbf{q} = -(k_r/\eta_r)\nabla p$. It is supposed that the fluid filtrating from the fracture into the reservoir has the same viscosity as the pore fluid. However, the Newtonian fluid within the fracture has different viscosity η_f. This corresponds to the normal situation in hydraulic fracturing when the fracturing fluid is a high-viscous gel and only its low-viscous base fluid is filtrated into the reservoir. However, for the sake of simplicity the filter-cake influence is not taken into account. We restrict ourselves to modelling the initial pad stage of hydraulic fracturing, when no proppant is added into the fluid. Thus, the transport of proppant along the fracture is not considered.

For the generality, the reservoir is initially subjected to a prestress with the stress tensor $\boldsymbol{\tau}^0(x, y)$. Since we observe only straight fractures, tensor $\boldsymbol{\tau}^0$ satisfies symmetry conditions relative to x-axis.

The governing equations of the quasi-static poroelasticity model are the following[20]:

$$\begin{aligned} &\operatorname{div}\boldsymbol{\tau} = 0, \quad \boldsymbol{\tau} = \boldsymbol{\tau}^0 + \lambda\operatorname{div}\mathbf{u}\,\mathrm{I} + 2\mu\,\mathscr{E}(\mathbf{u}) - \alpha p\mathrm{I}, \\ &S_\varepsilon\frac{\partial p}{\partial t} = \operatorname{div}\left(\frac{k_r}{\eta_r}\nabla p - \alpha\frac{\partial \mathbf{u}}{\partial t}\right). \end{aligned} \tag{1}$$

Here $\mathscr{E}(\mathbf{u})$ is the Cauchy's strain tensor $2\mathscr{E}(\mathbf{u})_{ij} = \partial u_i/\partial x_j + \partial u_j/\partial x_i\,(i, j = 1, 2)$, α is the Biot coefficient, $\lambda(\mathbf{x})$ and $\mu(\mathbf{x})$ are elasticity moduli, I is the identity tensor. The storativity S_ε reflects the dependence of the Lagrangian porosity ϕ on $\varepsilon = \operatorname{tr}\mathscr{E}$ and p as in[20]:

$$\frac{\partial\phi}{\partial t} = \alpha\frac{\partial\varepsilon}{\partial t} + S_\varepsilon\frac{\partial p}{\partial t}, \quad S_\varepsilon = \frac{(\phi_0 - \alpha)(1 - \alpha)}{K}, \tag{2}$$

where $K = \lambda + \dfrac{2\mu}{3}$ is the bulk modulus, ϕ_0 is the initial porosity. Due to the plane strain approximation, the solid phase displacement vector $\mathbf{u} = (u_1, u_2) = (u, v)$ is two-dimensional, and all vector operations are also taken in 2D space of independent variables $\mathbf{x} = (x_1, x_2) = (x, y)$.

Symmetry of the problem with respect to Ox-axis allows solving Eqs. (1) in domain $\Omega = \{(x, y){:}\ |x| \le R,\ 0 \le y \le R\}$ as shown in Fig. 1b.

Over the outer boundary $\Gamma_R = \{\partial\Omega \cap y > 0\}$ the confining far-field stress $\boldsymbol{\sigma}_\infty$ is applied and the constant pore pressure $p = p_\infty$ is prescribed:

$$\Gamma_R{:}\quad p = p_\infty, \quad \boldsymbol{\tau}\langle\mathbf{n}\rangle = \boldsymbol{\sigma}_\infty, \quad (\boldsymbol{\tau}\langle\mathbf{n}\rangle)_i = \tau_{ij}n_j. \tag{3}$$

Henceforth $\mathbf{n}$ and $\mathbf{s}$ denotes the outer normal and tangential unit vectors to the boundary of the domain Ω; the summation over the repeating indices is implied. We restrict ourselves to the case $\boldsymbol{\sigma}_\infty = -\sigma_\infty^{min}\mathbf{e}_2(\mathbf{n}\cdot\mathbf{e}_2) - \sigma_\infty^{max}\mathbf{e}_1(\mathbf{n}\cdot\mathbf{e}_1)$, where σ_∞^{max} and σ_∞^{min} are the maximal and minimal principal in situ stresses, respectively. Moreover, we assume that the prestress $\boldsymbol{\tau}^0$ satisfy the same boundary condition: $\boldsymbol{\tau}^0\langle\mathbf{n}\rangle|_{\Gamma_R} = \boldsymbol{\sigma}_\infty$.

In order to close the model it is supplemented with the initial data at some moment t^0:

$$\mathbf{u}|_{t=t^0} = \mathbf{u}^0(x, y), \quad p|_{t=t^0} = p^0(x, y), \quad L_i|_{t=t^0} = L_i^0, \ \ i = \ell, r. \tag{4}$$

2.2. Equations for the hydraulic fracture

The line $y = 0$ is divided into the part $\Gamma_f = \{-L_\ell(t) \le x \le L_r(t),\ y = 0\}$ occupied by the fracture, and the remaining part $\Gamma_s = \{-R < x < -L_\ell(t),\ y = 0\}\bigcup\{L_r(t) < x < R,\ y = 0\}$. Outside the fracture on Γ_s the symmetry conditions (see[15]) are satisfied:

$$\Gamma_s{:}\quad \frac{\partial u}{\partial y} = 0, \quad v = 0, \quad \frac{\partial p}{\partial y} = 0. \tag{5}$$

With $p_f(t, x)$ standing for the fluid pressure inside the fracture and σ_{coh} denoting the cohesive forces near the fracture's tips (explained below), the force balance over the fracture's wall yields

$$\Gamma_f{:}\quad p = p_f, \quad \mathbf{n}\cdot\boldsymbol{\tau}\langle\mathbf{n}\rangle = -p_f + \sigma_{coh}, \quad \mathbf{s}\cdot\boldsymbol{\tau}\langle\mathbf{n}\rangle = 0. \tag{6}$$

Here we neglect the tangential stress due to the fluid friction on the fracture's walls in comparison with the normal stress.

The fluid flow in the fracture is governed by the mass conservation law complemented with the Poiseuille formula:

$$\frac{\partial w}{\partial t} + \frac{\partial(wq)}{\partial x} = -q_l, \quad w \equiv v|_{y=0}, \quad q = -\frac{(2w)^2}{12\eta_f}\frac{\partial p_f}{\partial x}. \tag{7}$$

Here w is a half of the fracture aperture, q is the fluid velocity in x-direction. No fluid lag is assumed at the fracture's tip.

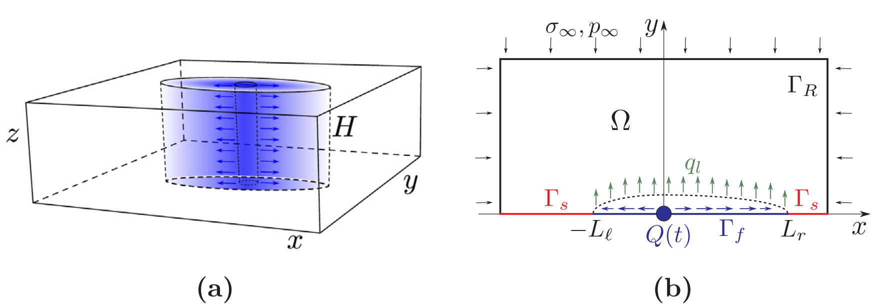

Fig. 1. Planar vertical hydraulic fracture in a poroelastic medium: (a) 3d view; (b) horizontal cross-section by plane $z = 0$.