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Three-dimensional strength estimation of intact rocks using a modified Hoek-Brown criterion based on a new deviatoric function

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ABSTRACT

Hoek-Brown criterion has been applied widely in a large number of rock projects around the world and has the main advantage that its input data can be determined from the uniaxial compressive strength, mineralogy and structural properties of the rock. However, there also exist demerits that neglect the effects of the intermediate principal stress and represent a non-smoothness shape in the deviatoric plane. After reviewing the history of the development of the Hoek-Brown criterion briefly, a modified Hoek-Brown criterion was proposed by the combination of a versatile deviatoric function and a meridian function of the original Hoek-Brown criterion. The new deviatoric function, which was proposed by introducing an extra strength parameter α into the one of Matsuoka-Nakai (MN) criterion and behaviors as well as the one proposed by Bigoni and Piccolroaz, is a generalization of deviatoric shapes belonging to several classic strength criteria, i.e. Drucker-Prager (DP), Tresca, Mohr-Coulomb (MC), Lade-Duncan (LD), Matsuoka-Nakai (MN) and Ottosen strength criterion. The procedure for determination of the strength parameters was demonstrated in detail. Comparisons between the derived criterion and true triaxial test data of KTB amphibolite, Westerly granite, Dunham dolomite, Mizuho Trachyte were presented in three-dimensional principal stress space qualitatively and quantitatively. Overall predicted errors were also compared with the ones obtained by the generalized Zhang-Zhu criterion (Z-Z) as well as its modifications (S-D, E-D, H-D), which showed least error for each rock type by the modification in this paper. The effect of the intermediate principal stress was also discussed for these four rock types, confirming that the intermediate principal stress is included in the strength criterion. The derived criterion not only maintains the features in the meridian plane of the original Hoek-Brown criterion, but solves the non-smoothness problem in the deviatoric plane and shows an excellent performance for describing brittle and ductile rock types considered.

1. Introduction

Many different strength criteria for rock materials and rock masses have been proposed in rock engineering practice.[1] The Hoek–Brown strength criterion has been used most widely among these strength criteria for its simplicity in formulation.

The Hoek-Brown criterion was originally proposed by Hoek and Brown[2,3] for use under the confined conditions surrounding underground excavations. Over the past three decades, the Hoek–Brown criterion has been developed into different versions to meet new applications and to deal with unusual conditions in rock engineering.[4] However, Hoek-Brown criterion does not take account of the influence of the intermediate principal stress, which has substantial influence on the strength of rock materials indicated by numerous experimental tests[5–10] and numerical tests.[11–13] Moreover, there exist singularities on the failure surface at triaxial compression and extension states.

Therefore, researchers have developed many different three-dimensional versions of the Hoek-Brown criterion, including Pan-Hudson criterion,[14] generalized Priest criterion[15], (generalized) Zhang-Zhu criterion,[16,17] Melkoumian criterion,[18] etc. Among these 3D criteria, none can predict the same strength as the original Hoek-Brown criterion at both triaxial compression and extension states except for the Zhang-Zhu criterion[16] and generalized Zhang-Zhu criterion,[17] which, however, still suffer from the non-smoothness and non-convexity problems leading to inconvenience in numerical applications. Such limitations can be overcome by replacing the Lode dependence with a smooth and convex one.[19,20]

A new two-parameter deviatoric function with smoothness and convexity characteristics, by revising the deviatoric function of the Matsuoka-Nakai criterion, was proposed for defining the shapes of the deviatoric plane, which has the same ability of shape variations as the one introduced by Bigoni and Piccolroaz,[21] covering deviatoric

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functions of Drucker-Prager, Tresca, Mohr-Coulomb, Lade-Duncan,[22] Matsuoka-Nakai[23] and Ottosen[24] strength criterion. On this basis, a modified Hoek-Brown criterion for rock was proposed as the product of two distinct functions, the new two-parameter deviatoric function and the meridian one of the original Hoek–Brown strength criterion.

The modified Hoek-Brown criterion has four strength parameters m_i, σ_c, α, A. Several methods, including rules-of-thumb,[25,26] grid search method,[9] least squares,[16,19] can be utilized to determine the first two strength parameters (m_i, σ_c), which belong to the meridian function of Hoek-Brown criterion. The latter two (α, A) controlling the convexity and smoothness of the deviatoric shapes, respectively, can be calculated by two steps.A is a pressure-dependent strength parameter and can be determined in the same way as the Matsuoka-Nakai criterion, which facilitates the derived criterion to contain pressure dependence and hence to describe the variation of deviatoric shapes from curved triangle to circle as hydrostatic pressure increases. Then, α can be estimated according to the deviatoric function with known A.

Finally, triaxial test data of brittle (KTB amphibolite, Westerly granite) and relatively ductile (Dunham dolomite, Mizuho Trachyte) rock types were selected from available literature to verify the derived criterion in three-dimensional principal stress space qualitatively and quantitatively. Results show that, the derived criterion considering the influence of the intermediate principal stress not only maintains the non-linear features in the meridian plane of the original Hoek-Brown criterion, but solves the non-smoothness problem in the deviatoric plane and has the incomparable advantages. Therefore, findings in this paper have significant senses in the research of rock strength criterion.

2. Description of the failure envelope in terms of Haigh–Westergaard representation

2.1. Stress state in the principal stress space and deviatoric plane

The Haigh–Westergaard representation[27] is employed as the analysis is restricted to isotropic behavior in the present paper. Besides, compressive stress is assumed to be positive due to the fact that the stress is compressive in most cases in rock mechanics.

Fig. 1 is the diagram of Haigh–Westergaard coordinate system. Any stress point P in the principal stress space can be conveniently represented by a stress tensor σ_{ij}. Three principal stresses are equal in the hydrostatic axis, the plane perpendicular to which is defined as the deviatoric plane (also called octahedral plane or π plane). The function

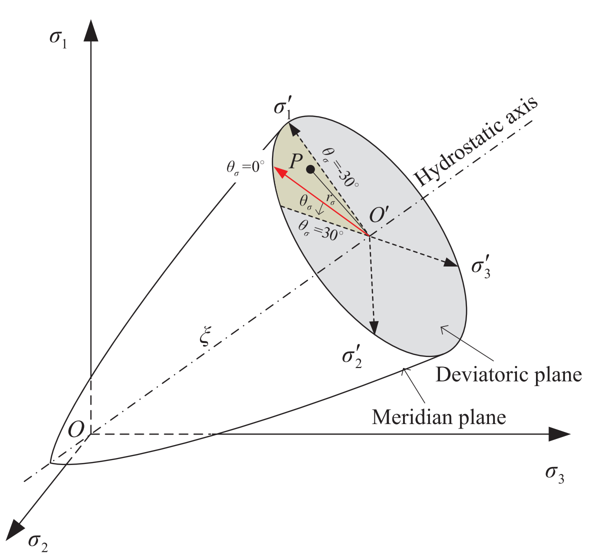

Fig. 1. Schematic of a typical failure envelope for isotropic pressure sensitive material in principal stress space.

describing the failure curves in the deviatoric plane can thus be referred to as Lode dependence or deviatoric function. The invariant ξ is the hydrostatic component of stress point P, defined as the distance of the deviatoric plane to the origin O of principal stress space. In the deviatoric plane, the sum of three principal stresses is a constant I_1.r_σ is the radial distance of stress point P to the hydrostatic axis, representing the magnitude of deviatoric stress.θ_σ is known as the Lode angle, which is the measure of rotation from x axis perpendicular to projected σ'_2 axis (y axis). The Lode angle in this paper ranges from -30°(triaxial compression, $\sigma_2 = \sigma_3$) to 30°(triaxial extension, $\sigma_1 = \sigma_2$).

According to the notion of stress invariants,[28] it is convenient to relate the parameters above (ξ, r_σ, θ_σ) to the Haigh–Westergaard invariants (I_1, J_2, θ_σ) as

$$\left.\begin{aligned} \xi &= \frac{I_1}{\sqrt{3}} \\ r_\sigma &= \sqrt{2J_2} \\ \theta_\sigma &= \frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right) \end{aligned}\right\} \tag{1}$$

where $I_1 = \sigma_{ii}$ is the first invariant of the stress tensor σ_{ij}, $J_2 = s_{ij}s_{ij}/2$ and $J_3 = s_{ij}s_{jk}s_{ki}/3$ is the second and third invariant of the deviatoric stress tensor $s_{ij} = (\sigma_{ij} - \sigma_{kk}\delta_{ij}/3)$, respectively.

Then, the major, intermediate and minor principal stresses, σ_1, σ_2 and σ_3, can be derived as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \sqrt{\frac{2}{3}}\, r_\sigma \begin{Bmatrix} \sin\left(\theta_\sigma + \frac{2}{3}\pi\right) \\ \sin\theta_\sigma \\ \sin\left(\theta_\sigma - \frac{2}{3}\pi\right) \end{Bmatrix} + \begin{Bmatrix} \frac{I_1}{3} \\ \frac{I_1}{3} \\ \frac{I_1}{3} \end{Bmatrix} \tag{2}$$

2.2. Requirements for Lode dependence for geomaterials

Lode dependence, described by a deviatoric function $g(\theta_\sigma)$, is a key in controlling the shape of the failure surface and implies the effect of the third invariant on geomaterials. Strength criteria generally obey the requirements for aspect ratio, differentiability and convexity,[29] which can be further used to develop a plasticity-based constitutive model for geomaterials.

Firstly, the aspect ratio is defined as

$$K = g(30^\circ) \tag{3}$$

which varies between 0.5 and 1 for geomaterials,[22,23] which controls the general shape of π plane. It should be noted that $g(-30^\circ)$ is set to 1 for normalization.

Then, the deviatoric function $g(\theta_\sigma)$ is required to be differentiable at $\theta_\sigma = \pm 30^\circ$, which means

$$g'(\pm 30^\circ) = 0 \tag{4}$$

The requirement on convexity of the deviatoric function in the deviatoric plane comes from the experimental[30,31] and theoretical[21,32] results. The convexity requirement can be expressed as

$$g^2 + 2g'^2 - gg'' \geq 0 \tag{5}$$

3. Hoek-Brown criterion and characteristics of its failure envelope

This section describes the brief development history of the Hoek–Brown criterion and the characteristics of the Hoek–Brown strength criterion in 3D principal stress space from the aspects of deviatoric plane and meridian plane.

3.1. A brief development history of the Hoek-Brown criterion

The Hoek-Brown criterion was developed in the late 1970s to provide input for the design of underground excavations.[2,3] The original