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Insights on rock fracture from digital imaging and numerical modeling

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ABSTRACT

Rock fracture and the induced displacement field are investigated through physical and numerical experiments. A Berea sandstone specimen with a central notch was tested in three-point bending. Digital image correlation (DIC) was used to obtain the displacement patterns as the fracture initiated and propagated. The DIC results clearly show the development of a damage zone and displacement discontinuity along the center line of the beam at peak load. It is suggested that a traction-free or cohesionless crack may exist before the unstable growth, a condition normally ruled out in the literature based on numerical modeling within the frame work of a cohesive zone finite element analysis. In this note, the displacement profiles determined from DIC are compared with those from numerical simulations using (i) nonlinear finite element and (ii) bonded particle models. It is shown that the bonded particle model is capable of reproducing the physical displacement field with better accuracy compared to that from the finite element model with softening. Furthermore, as supported by experiments and bonded particle simulations, it appears that a cohesionless crack can develop at peak load, an observation that is not confirmed by the nonlinear finite element analysis.

1. Introduction

Prediction of fracture is an important problem in rock mechanics to assure the safety and stability of underground structures or to promote efficient drilling and excavation. Because rock is not an ideal brittle material, a significant nonlinear region near the crack tip called the fracture process zone (FPZ) causes deviation of behavior from that predicted by linear fracture mechanics.[1–4] To provide a simplified theory of fracture, the FPZ is often considered as an equivalent crack, where the length of the actual crack together with a portion of the FPZ is imagined as the total length of the crack, which is used to obtain a better estimate of the fracture energy of the material. To simulate the FPZ, a cohesive crack model is sometimes used in which a law for the stress-crack face separation needs to be defined.[1] However, a consequence of the cohesive zone model within a local finite element simulation is that there is no cohesionless (traction-free) crack at peak load.[5]

In this note, using a bonded particle discrete element system together with a physical three-point bending test, we show evidence for the possibility of a traction-free length plus FPZ at peak load. Digital image correlation (DIC) is used to measure the displacement patterns of the physical specimen. In addition to the bonded particle model, nonlinear fracture analyses are conducted to simulate the rock structure. The results of the different numerical simulations are compared with the DIC measurements and the existence of a cohesionless crack at peak load is investigated and discussed.

2. Digital image correlation

Digital image correlation was introduced to solid mechanics in the 1980s.[6] With the growth of digital cameras and efficient correlation algorithms, DIC proved to be a powerful experimental technique for various applications, especially in fracture of rock.[7–9] Intuitively, DIC is a technique that simply traces a unique particle on the specimen surface to obtain displacements. The unique particle, a subset with several speckles, serves as a target to determine its movement between two images.[9,10] If the experiments involve small deformation, it implies that the subset does not exceed a certain range of movement and does not undergo a large change of its size and shape. A relatively small area, the region of interest (ROI), can be selected as a searching area for a subset. Due to displacement, an $m \times m$ subset (m = number of pixels) centered at position P moves to position p within the $n \times n$ ROI during the experiment, where $n > m$. The subset of P is actually from a reference image and the subset of p is from a current image, but they share a

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global coordinate system (X, Y). The DIC algorithm identifies these two subsets between reference and current images and then determines the displacement (u,v). Thus, the evaluation of similarity between the different subsets becomes essential and a physical criterion needs to be proposed for their identification. This can be achieved by a cross-correlation with a Fast Fourier Transform (FFT) method. The cross-correlation function CC is defined as the two-dimensional spatial convolution of I and I^*:

$$CC = \sum_{i=1}^{m}\sum_{j=1}^{m} I(x_i, y_j)I^*(x_i^*, y_j^*) \tag{1}$$

where I and I^* are intensity values from reference and current images, respectively. According to the optical speckle phenomenon, the peak position of cross-correlation values CC represents the moved position of the deformed subset.[11,12] In other words, there exists a maximum value of the cross-correlation function for a given subset in the ROI, such that it satisfies the conditions $x^* = x + u$ and $y^* = y + v$. Then the subset displacement (u,v) is determined.

3. Experimental setup

The tested material, Berea sandstone, consists of uniformly sized grains from 0.1 to 0.8 mm, with the average grain size of 0.2 mm. The material properties are porosity = 20%, Young's modulus (uniaxial compression) = 10.0 GPa, Poisson's ratio = 0.3, tensile strength (modulus of rupture) = 3.4–3.6 MPa, and uniaxial compressive strength = 28–32 MPa. A beam under three-point bending (3PB) was used to achieve an opening mode fracture. Crack initiation and propagation were controlled by a closed-loop, servo-hydraulic system with crack mouth opening displacement (CMOD) as the feedback signal. The dimensions of specimen CN5-1 were height H = 62.46 mm, span S = 146.8 mm, thickness B = 30.9 mmm, and notch length a = 3.44 mm, a/H = 5.0%.

Specimen preparation of the speckles should satisfy the basic DIC principle, which is the existence of a unique particle, actually the subset of unique speckle patterns, in the reference and current images. Two features should be considered:[1] distribution of speckles in the pattern and[2] average size of the speckles. Well-distributed speckles will help to increase the contrast of the speckle pattern and decrease the measurement bias and noise. The average size of the speckles fundamentally influences image sampling and selection of subset size. For a well-prepared speckle pattern, the average size of speckles is about five pixels. A subset generally consists of 6–9 random speckles, which have various sizes and various intensity values.[8,9]

The DIC system consists of a Unibrain (San Ramon, CA) Fire-I 810 IEEE-1394 CCD camera with 1600 × 1200 effective square pixels. Here a magnification factor M is introduced to transform the results from the digital image to the physical dimension on the specimen surface, because the matching process is performed on intensity values of the digital images and the “unit” of measure from the computational analysis is pixel-based. Considering the importance of the field of view and the fixed CCD array of the digital camera, the magnification factor M = 30.0 μm/pixel with the observation area of 36 × 48 mm. The measurement resolution of the DIC system is on the order of 0.3 μm[8,9].

4. Bonded particle model and tests results

4.1. Bonded particle model

The bonded particle model (BPM) is a type of discrete element method for simulation of materials composed of grains. This method was successfully used for simulation of soil particles interactions by Cundall and Strack.[13] Further, by allowing the particles to stick to each other at the contact points, a solid material such as rock or concrete can be modeled. This bonded particle model (BPM) has been used extensively in the literature for simulation of geomaterials.[14–18]

The micromechanical parameters for the interaction of two circular particles (disks) include normal spring stiffness K_n, shear spring stiffness K_s, normal bond n_b, shear bond s_b, and friction coefficient μ. The hydrostatic stress used to create small overlaps of the particles during the sample preparation called the genesis pressure is also needed. To control the softening response or rock ductility, the normal bond between particles is allowed to follow a linear softening path; once the contact force between two disks reaches its maximum value n_b, the contact force follows a linear reduction due to additional opening or extension. The slope of the normal contact force-normal displacement curve in the post-peak regime is called K_{np}, which is an additional micromechanical parameter for the model. By decreasing the K_{np} value (for a fixed K_n, increasing the K_n/K_{np} ratio) the material shows more ductile behavior. The particle contacts at the initial stage of loading behave elastically. Following the complete loss of the contact tensile or shear strength, the contact follows a Coulomb frictional behavior with the friction coefficient μ if the contact is under compression, otherwise the two particles involved with the contact detach completely from each other. The computer program CA2,[19] a hybrid discrete-finite element code for 2D simulation of geomaterials, is used for the numerical analysis.

The BPM was calibrated,[20] and as a result, the following micromechanical parameters were obtained: K_n = 10 GPa, K_s = 2.5 GPa, n_b = 69 N/m, s_b = 690 N/m, K_{np} = 0.05 GPa (K_n/K_{np} = 200) and μ = 0.5. The particles’ radii were within the range of 0.068–0.082 mm and a genesis pressure of 0.92 GPa was used for sample preparation. Very small size particles and a high ductility value (K_n/K_{np} = 200) were needed to capture the extensive pre-peak inelasticity (ductility) observed in the sandstone. These micro-mechanical parameters were used in numerical uniaxial compression (40 × 80 mm specimen) and three point bending (160 × 63 mm beam with the span of 147 mm) tests and the following mechanical properties were obtained: Young's modulus = 7.0 GPa, Poisson's ratio = 0.25, UCS = 31.6 MPa, and modulus of rupture = 3.8 MPa. These values are in good agreement with the physical properties of the rock. Note that the Young's modulus was intentionally calibrated for a smaller value compared to the physical data as the modulus in tensile loading is smaller than that in compression loading due to microcracks that open.

4.2. Tests results

A three point bending test on a numerical specimen with the same dimensions as the physical specimen CN5-1 was conducted. A total of 633,000 particles were used for the simulation of the beam with a center notch of 3.4 mm length and 2 mm width. The crack (notch) mouth opening displacement (CMOD) was recorded in the numerical simulation. The experimental and numerical responses are compared in Fig. 1. Note that the peak loads for the numerical and experimental beams are showing close agreement, 1498 N and 1473 N, respectively. The initial slopes of the curves in Fig. 2 are also showing a good match, indicating the Young's modulus of 7 GPa is an appropriate value to capture the elastic behavior of the rock beam.

The incremental horizontal or x-displacement profiles along horizontal lines were studied around the central part of the beam. The incremental horizontal displacement for a load range of 90–100% of the peak load are considered. Fig. 2a–d compare the physical (from DIC measurements) and numerical data along horizontal lines located at different y-values. A relatively good match between the experimental and numerical data is observed. In particular, close to the notch tip at y = 3.9 and 8.7 mm, a sharp jump, an opening displacement discontinuity, of several microns is observed when the center line of the beam is crossed, suggesting severe damage at these locations. On the other hand, at greater y values (y = 15.9, 30.3 mm), a smooth transition in the horizontal displacement is realized, suggesting less or no damage at these locations.

The increments of the vertical displacement for the loading interval