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Effect of impact velocity, block mass and hardness on the coefficients of restitution for rockfall analysis

Pavlos Asteriou*, George Tsiambaos

National Technical University of Athens, Greece

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ABSTRACT

One of the most significant input parameters in rockfall trajectory modelling is the Coefficient of Restitution, which controls block rebound. In particular, in the design of rockfall barriers, it can significantly affect their height. In practice, the coefficients of restitution are acquired by field or laboratory tests, back analyses of known events, experience or, most commonly, suggested values connected to the slope material. Furthermore, to account for the effect of impact velocity or block mass, scaling factors are available. However, the suggested values are based on a qualitative description of the slope material, and the scaling factors are not well documented, nor do they account for the effects of impact velocity and block mass simultaneously. In this paper, a semi-empirical correlation is proposed that takes into account the Schmidt hammer rebound value of both the slope and block material as well as the impact velocity and block mass. This was derived by an extensive laboratory experimental study (445 impact tests) of a one-dimensional drop of spherical blocks onto planar surfaces. The semi-empirical correlation proposed adequately describes the responses observed under the circumstances imposed in the laboratory.

1. The coefficient of restitution

The coefficient of restitution (*COR*), or the rebound coefficient (*R*), is defined as the decimal fractional value that represents the ratio of velocities before and after an impact of two colliding entities. Theoretically, a *COR* that equals one corresponds to a perfect elastic collision, a *COR* less than one defines an inelastic collision and *COR* of zero describes the instantaneous stopping of the block at the surface area, without rebound, i.e., a perfectly plastic impact.[1] However, in practice, *COR* will be less than one, even in elastic normal impacts, as argued by Imre et al.[2]

After an impact, the block velocity changes according to the *COR* value. Hence, in rockfall engineering practice, *COR* is assumed to be an overall value that takes into account all the characteristics of an impact, including deformation, sliding at the contact area, and transformation of rotational moments into translational ones, and vice versa.[3] Moreover, *COR* is influenced by a wide range of parameters, as summarized in Table 1.

Various definitions on the *COR* s have been proposed, such as the kinematic, kinetic and energy *COR*. However, there is no consensus on which of them is more appropriate for modelling the impact during a rockfall.[5]

1.1. Definitions for the coefficients of restitution

The kinematic approach, derived from the inelastic collision of particles in Newtonian mechanics, is the first and simplest definition. For an object (block) impacting on a steadfast surface (slope), kinematic *COR* (v_{COR}) is defined as

$$v_{COR} = \frac{v_r}{v_i} \tag{1}$$

where v_i is the incident velocity magnitude and v_r is the rebound velocity magnitude (see Fig. 1)

In engineering practice, the most commonly used *COR* definition is derived from the normal and tangential projections of the kinematic *COR* to the impact surface. Normal to the slope surface, energy dissipation is associated with the deformation of the colliding entities and the propagation of elastic waves. Normal *COR* (n_{COR} also found as R_n, k_n) is determined by

$$n_{COR} = -\frac{v_{n,r}}{v_{n,i}} \tag{2}$$

In the tangential direction, energy losses are attributed to the friction between the colliding bodies and to the transformation of rotational moments into translational ones, and vice versa. Therefore,

* Corresponding author.
E-mail address: paster@central.ntua.gr (P. Asteriou).

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Table 1
Parameters that influence the rebound response.[4]

Slope characteristics	Block characteristics	Kinematics
Strength	Strength	Translational velocity
Stiffness	Stiffness	Rotational velocity
Roughness	Weight - Size	Collision angle
Inclination	Shape	Configuration of block

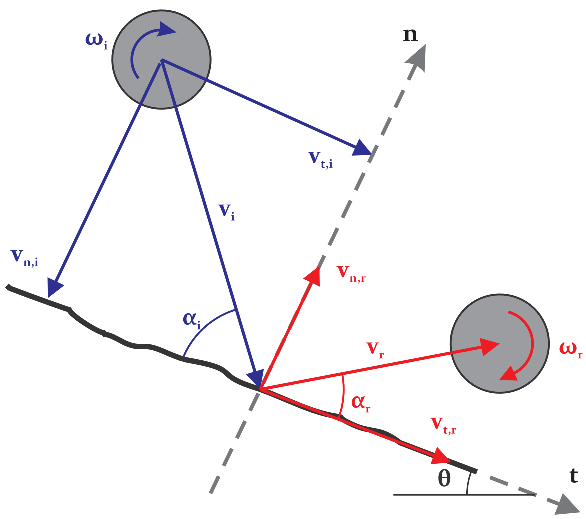

Fig. 1. Quantities used in COR definitions.

tangential *COR* (t_{COR}, also known as R_t, k_t) is defined as

$$t_{COR} = \frac{v_{t,r}}{v_{t,i}} \tag{3}$$

where the subscripts n and t in Eqs. (2) and (3) denote the normal and tangential velocity components with respect to the slope surface (see Fig. 1).

Another definition of *COR* found in the literature is given in energy terms and is known as Energy *COR* (E_{COR}):

$$E_{COR} = \frac{0.5(mv_r^2 + I\omega_r^2)}{0.5(mv_i^2 + I\omega_i^2)} \tag{4}$$

where m is the mass, I is the moment of inertia and ω is the angular velocity magnitude of the block.

However, in relevant studies, Energy *COR* (Eq. (4)) is mostly found without considerating the rotational motion, according to Eq. (5). The latter is hereafter denoted e_{COR} to distinguish it from Eq. (4):

$$e_{COR} = \frac{0.5mv_r^2}{0.5mv_i^2} \tag{5}$$

It is apparent that e_{COR} is equal to the square root of v_{COR}. Therefore, this definition does not provide any benefit compared to the simpler definition of v_{COR}.

Additionally, more complex definitions have been proposed, such as the impulse *COR* s defined by Descoeudres et al.[6] and Pfeiffer and Bowen.[7] However, they are rarely used due to their overall complexity.

1.2. Characteristic COR values

In practical applications, the *COR* values are selected based on the material that constitutes the slope surface. For that purpose, some indicative values are provided as guidelines. Table 2 summarizes the values suggested by Hoek[8], which have been adopted in the RocFall simulation software[9] with a standard deviation of 0.04. Similar values were initially proposed by Pfeiffer and Bowen[7]. A detailed documentation of the *COR* values obtained from experimental studies can be found in Heidenreich[10], Turner and Schuster[11], Asteriou[12] and other studies.

Table 2
Characteristic normal and tangential COR values.[8]

Slope Properties	normal COR	tangential COR
Clean hard rock	0.53	0.99
Asphalt road	0.40	0.90
Bedrock outcrops with boulders	0.35	0.85
Talus cover	0.32	0.82
Soft soil with vegetation	0.30	0.80

The aforementioned values are suggestive. Software developers encourage designers to perform field tests and back-analyse the trajectories to acquire site-specific values. Unfortunately, in most cases, this procedure is omitted because it is a difficult, expensive and time-consuming task. Therefore, a design is often based on the suggested values.

However, the *COR* values originating from laboratory and field tests or back analyses of rockfall events are significantly scattered and do not show any dominant or systematic pattern. This is depicted in Fig. 2, where the aforementioned values are compared to those suggested in the literature for the case of a hard rock. Suggested *COR* is plotted by the blue circle (the centre is the suggested value (n_{COR}, t_{COR}), and the radius is the standard deviation), and the data originating from the relevant literature[13,8,7,14,3,15–20,5,21,10,22,2,23–25,11,26–28,12] are plotted by red points.

The rebound height is very sensitive to the selected value, as observed in Fig. 3 through a simple parametric analysis altering n_{COR}. By contrast, the kinetic energy is not as strongly affected; in a practical application, the proposed barrier would have the same capacity. Moreover, selecting a *COR* based solely on the material type erroneously omits the effects of the parameters given in Table 1.

1.3. Effect of rock type on COR values

As a general rule, *COR* increases as the slope material becomes "harder". This is qualitatively verified by the suggested values presented in Table 2 but becomes questionable when using values determined experimentally or in back-analysis studies, as shown in Fig. 2. This difference is attributed to the highly volatile nature of the

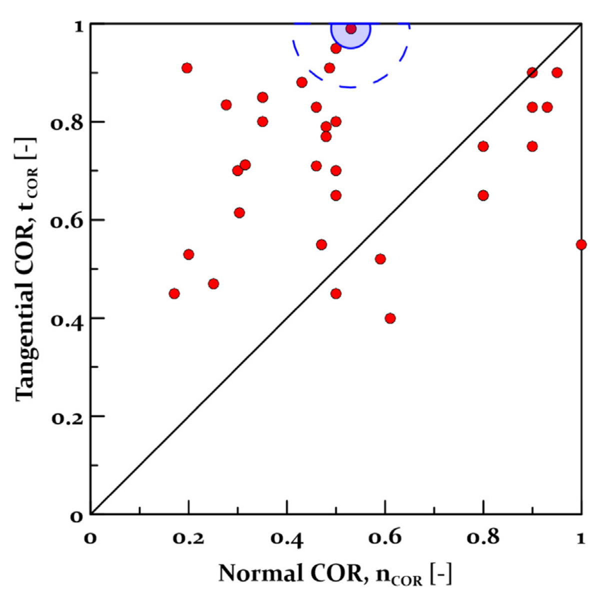

Fig. 2. Comparison of suggested *COR* (blue circle) with the values obtained from experimental studies for hard rock slopes (red points).