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A modified extended finite element method for fluid-driven fractures incorporating variable primary energy loss mechanisms

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ABSTRACT

A coupled extended finite element method (XFEM) is presented here for modeling propagation of fluid-driven fractures in different regimes including toughness- and viscosity-dominated regimes. The extended finite element method allows to model growth, and coalescence of arbitrary discontinuities (fractures) without requiring the mesh to conform to discontinuities nor significant refinement near the fractures. Fluid-driven fractures propagation is a coupled, nonlinear and non-local problem with moving boundary conditions. The proposed method is based on the extended finite element method with modifications to incorporate variable stress singularity at the crack tips for the transition between toughness-dominated and viscosity-dominated regimes. These modifications consist of enriched functions that are initially inspired by the asymptotic analytical solutions. The standard extended finite element approximation is enriched by adding near tip asymptotic solutions just for displacements, however the proposed method introduces a consistent enriched function for fluid pressure calculations close to the fracture tips to catch the singularity. Additionally, a technique is presented to remove singularity issue for required numerical integrations. Green's functions concept is proposed here to expedite calculations. To circumvent violation of partition of unity and parasitic terms in the approximation space induced by the blending elements at the edge of the enriched domain, the ramp function is utilized to improve the convergence rate. Stress intensity factors are calculated using a new contour integral method that can handle cases with different tip singularities. The proposed technique is verified with the cases that have analytical solutions. Some examples are presented to show the advantages of this technique in comparison to the regular XFEM.

1. Introduction

During the last two decades, a gigantic amount of natural gas has been discovered in low-permeability reservoirs around the world. Because of the low permeability of these formations and the low conductivity of the natural fracture networks, stimulation techniques such as hydraulic fracturing are necessary to make economic production possible.[1] The low conductivity of the natural fracture system could be caused by occluding cements that precipitated during the digenesis process.[2] The fact that natural fractures might be sealed by cements does not mean that they can be ignored while designing well completion processes. Cemented natural fractures can still act as weak paths for fracture growth. Hence, the presence of natural fractures adds more complexity to the induced fracture geometry, which may make fracturing jobs very challenging due to the formation of complex fracture geometries.[3] Nevertheless, even for simpler fracture geometries many current commercial software packages do not consider coupling between the rock deformation and fluid flow inside the fracture. To cover this shortage, the proposed method solves fluid flow and rock deformation equations at the same time. Modeling complicated fracture pattern developments is also possible within the context of the method presented in this paper.

Hydraulic fracturing is a common technique not only for enhancing hydrocarbon production, but also for improving geothermal energy extraction.[4] It is also widely used for other purposes like hazardous solid waste disposal,[5] measurement of in-situ stresses,[6] and fault reactivation in mining.[7] Hydraulic fractures, which are naturally induced by pressurized fluid in the host rock, are also observed in outcrops as joints,[8] veins,[9] and magma-driven dikes.[10] Hydraulic fracturing is also identified as a main mechanism in hydrocarbon migration through low permeable cap rock.[11]

A primary difficulty of accurate modeling hydraulic fracturing problems comes from the coupling of the fluid flow inside the fracture and rock deformations, which provide the fracture's width. Inaccurate

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calculations can lead to hazardous consequences, for instance, excessive fluid pressure may lead to creating a cylindrical crack around the wellbore.[12] Unfortunately, analytical solutions for fracture propagation[13] are only limited to simple geometries and limiting assumptions such as homogenous or isotropic medium. In the general case, modeling fluid-driven fractures is tremendously difficult even for simple geometries.[14] This difficulty is due to moving boundary conditions as a result of crack tip propagation, non-linearity of the governing equation for fluid flow in fractures, the high displacement gradient near the fracture tips, and non-locality of the solution. Non-linearity comes from the fact that fracture permeability is a cubic function of the fracture width. Non-locality means that the fracture width at one point is a function of fluid pressure at another point along the fracture.[13]

In recent years, there has been a return to analytical solutions to achieve a better understanding of different fracture propagation regimes. Analytical solutions are limited to very simple planar geometries in a homogeneous isotropic medium, but they provide insight into asymptotic behavior of the fluid pressure distribution near the fracture tips. Analytical solutions have revealed the controlling role of two energy dissipative processes in impermeable formations: fracturing of rock (toughness) and dissipation in the fracturing fluid due to friction (viscosity).

Depending on competition between the dissipative processes, the singularities at the fracture tips may vary. Adachi[15] introduced dimensionless toughness $\widetilde{K}$ or dimensionless viscosity M as the controlling parameter to determine whether fracturing the rock or friction losses is the primary energy loss mechanism during hydraulic fracture's propagation

$$\widetilde{K} = \frac{K'}{(E'^3\mu' Q_0)^{1/4}} = M^{-1/4}, \tag{1}$$

where K' and μ' are material parameters that are defined as $K' = 4\,(2/\pi)^{1/2}K_{Ic}$ and $\mu' = 12\mu$, $E' = E/(1-\nu^2)$ is the plain strain modulus of elasticity, Q_0 is the injection rate, μ is the fluid viscosity, K_{Ic} is the fracture toughness of the rock, E is the Young modulus of the rock, ν is the Poisson's ratio of the rock. $\widetilde{K} < 1$ is represented the viscosity-dominated regime and $\widetilde{K} > 4$ is represented the toughness-dominated regime. Adachi[15] showed that stress singularity at the fracture tip varies from $r_t^{-1/2}$ to $r_t^{-1/3}$, where r_t is the distance from the fracture tip. Hence, any sophisticated numerical method should be able to adjust for a varying order of singularity (λ) at the fracture tip. The order of singularity is a function of fracturing fluid viscosity, fracture toughness of the rock, and fracture length.[15] The exponent in r_t is the order of singularity minus one.

To address above-mentioned challenges, several numerical methods using the finite element methods,[16] the extended finite element method (XFEM),[17–19] continuum damage failure model,[20] cohesive zone model,[21] and the boundary element methods[22] have been proposed in the literature to model the hydraulic fracturing propagation. Furthermore, Dahi Taleghani[23] provided an overview of numerical models that can be used for modeling of interaction between hydraulic and natural fractures. In the finite element framework, modeling of crack growth has been carried out by applying various re-meshing strategies in the literature,[24] but re-meshing is computationally burdensome, involving transfer of data between different meshes. To address this inefficiency, the XFEM was developed.[17] In this approach, discontinuities such as fractures are allowed to propagate independently of the mesh configuration by permitting the discontinuity to cross the elements. For this purpose, finite element space will be enriched by additional functions, which are based on the analytical solution of the problem. Therefore, the new functions make it possible to embed discontinuities in the solution space. The enrichment is performed from a node to a node in a mesh by activating additional degrees of freedom when needed. A regular mesh for solving elasticity equations consists of quadratic triangular elements, i.e. each element has six nodes or twelve degrees of freedom. If the hydraulic fracture crosses a quadratic triangular element, two extra degrees of freedom per a main node of the triangular element associated with the sign function are activated. If a quadratic triangular element has the fracture tip inside, eight extra degrees of freedom per a main node of the triangular element are activated to honor tip enrichment. A regular mesh for solving fluid flow equations consists of quadratic linear elements, i.e. each element has three degrees of freedom. If an element has the fracture tip, an extra degree of freedom is activated to catch the singularity at the fracture tip for the viscosity-dominated regime. The XFEM has more advantages: the symmetry and sparsity of the stiffness matrix is preserved, the crack geometry can be completely arbitrary with respect to the mesh, and automatic enforcement of continuity.

Lecampion[25] attempted to use the XFEM to model the hydraulic fracturing problem. He sought the elasticity solution via the XFEM for a given fracture geometry with either specified fluid pressure distribution or fracture width. Modeling was limited to fractures located along the element edges; the fracture propagation and coupling process was not addressed in Lecampion.[25] The main advantage of the XFEM was neglected by assuming that the fractures were required to be aligned with the element edge. Gordeliy and Peirce made a significant effort to investigate the use of the XFEM for hydraulic fracturing problems. Gordeliy and Peirce[26] described coupled algorithms that use the Extended Finite Element Method (XFEM) to solve propagation of hydraulic fractures in an elastic medium. Gordeliy and Peirce[27] investigated convergence of the XFEM and presented novel enrichment basis functions.

We chose the XFEM to solve the hydraulic fracture propagation problem, because it particularly provides the opportunity to model fracture growth without mesh updating during the fracture network evolution. Furthermore, the XFEM does not require a high mesh concentration near the crack tip to catch the singular behavior of stress field at the crack tips.[17] The geometry of the fractures is handled in the model by the level set method, similar to.[28] Additionally, singularity of the fluid pressure at the fracture tips[29] is modeled by introducing extra enrichment for fluid pressure solution in the XFEM. To expedite numerical modeling process, Green's functions are used for fracture propagation calculations. Green's functions represent contribution of a unit load that is applied at a point along the fracture to the total stress intensity factor at the fracture tips.

2. Governing equations

The hydraulic fracturing process involves coupling three processes: (i) mechanical deformation of the formation caused by fluid pressure inside the fracture, (ii) fluid flow within the fracture network, and (iii) fracture propagation.[14] Rock deformation is usually modeled using linear elasticity. The fluid flow inside the fracture is typically simplified to flow along a channel by using lubrication theory. The fracture propagation process is considered in the framework of linear elastic fracture mechanics (LEFM) theory. Now, we will describe how all three processes are modeled in this paper.

The plane strain geometry is utilized in this paper. We consider a regular body Ψ bounded by a smooth curve Γ. The boundary Γ of the body Ψ can be divided into Γ_u and Γ_t, depending on whether the boundary conditions at a given location on the boundary are displacement (Dirichlet) boundary conditions (Γ_u) or tractions (Γ_t). Let u and ε be the displacement and strain field, respectively. The strong form, which is transformed into the weak form in Section 3 and solved, of the initial boundary value problem has the following form

$$\begin{aligned} &\sigma + b = 0 \quad \text{in } \Psi,\\ &u = \bar{u} \quad \text{on } \Gamma_{\mathrm{u}},\\ &\sigma\cdot n = \hat{t} \quad \text{on } \Gamma_{\mathrm{t}},\\ &\sigma\cdot n = P \quad \text{on } \Gamma_{\mathrm{cr}}, \end{aligned} \tag{2}$$