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A lined hole in a viscoelastic rock under biaxial far-field stress

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ABSTRACT

In this paper, we revisit the problem of an infinite, homogeneous, isotropic, linear viscoelastic medium containing a lined circular hole in the general setting of a biaxial far-field stress and for a variety of viscoelastic models for the medium and the lining. The goal is to provide reliable ready-to-use tool to analyze the plane-strain problem of a borehole in viscoelastic rock for any type of viscoelastic model that can be expressed in differential form. The lining is modeled as an isotropic viscoelastic ring. A constant biaxial loading is applied at infinity and/or a time-dependent pressure is prescribed at the inner boundary of the lining. Two distinct scenarios are considered: (i) the hole is excavated before the lining is inserted, i.e. the radius of the hole is larger than the outer radius of the ring, and (ii) the lining is set up directly and a perfect bond is assumed between the lining and the viscoelastic material. These problems are solved using the correspondence principle. The Laplace transformed stresses and displacements in both the lining and the rock are calculated. An inverse Laplace transform is then used to obtain the time domain solution. The operations of space integration and Laplace transform are performed analytically. The obtained new solution in the Laplace domain is compared with published results as well as a numerical solution by the finite element method (for the elastic transformed problem). The Laplace inversion procedure is done either analytically or numerically. For the sake of illustration, examples are given for the cases where the viscoelastic rock responds elastically in dilation and viscoelastically in shear. Five classical viscoelastic models in shear are notably considered. The accuracy of the approach for viscoelastic problems is demonstrated by comparing selected results with a few available analytical solutions for the simpler case of a hydrostatic load at infinity and an incompressible rock. We notably clarify the validity of a number of solutions described in the literature. The efficiency of the solution presented here is illustrated by several numerical examples. For reproducibility, a Mathematica script containing all the derived solutions is provided as supplementary material.

1. Introduction

Many applications in mining and petroleum engineering are related to the excavation and long term maintenance of tunnels and boreholes. To insure the structural integrity and mitigate the effects of the surrounding rock, the borehole or tunnel is typically supported by a lining (concrete or steel casing) inserted after the completion of the excavation. The knowledge of the long term behavior of such structures is, therefore, of interest for engineering purpose. Experimental observations suggest[1] that at small strains, low temperature and pressure, the assumption of a linearly viscoelastic response of rocks is reasonable. It is also reasonable to suggest that (i) the excavation is performed at a sufficiently small rate such that inertial effects are negligible, (ii) the length of the borehole or tunnel is significantly longer than its cross-section, and (iii) the cross-section is of circular shape. The problem involving a tunnel or borehole can thus be mathematically modeled as the plane strain problem of an infinite, isotropic, homogeneous, linearly viscoelastic medium containing a lined circular cavity. The lining can be modeled as an isotropic elastic or viscoelastic ring. This particular problem has already been extensively studied both numerically or analytically in the literature. However, most relevant publications either make the simplifying assumption of a hydrostatic far-field stress[2–7] or use very simple viscoelastic models.[8] In addition, the majority of the reported closed-form solutions are obtained by adopting the even stronger assumption of an incompressible rock material.

In the present paper, we re-investigate the problem in a more general way. We notably consider that the initial far-field stresses are truly bi-axial (i.e. with a deviatoric component). In the case where we account for the presence of a lining, we assume that the lining is put in place sufficiently quickly or/and that the deviatoric component of the far-field stress are smaller than the hydrostatic component such that the excavation remains circular. This is an idealization as when deviatoric

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components are present, the shape of the borehole changes with time to an ellipse and/or the contact between the lining and the rock could be, in general, imperfect. In addition, the assumption of the instantaneous installment of the lining might lead to overestimation of the lining pressure at early times only, as discussed in the literature.[9] However, these simplifying assumptions are still acceptable for practical rock engineering problems and the solutions obtained here can be viewed as two asymptotic models of the construction process: i) no lining in place, ii) lining put in place instantaneously during excavation. An important novelty of the solution presented here is that it can be applied to a wide class of viscoelastic models that can be described in terms of combination of springs and dashpots. It also does not make the assumption of an incompressible material a-priori. Our approach and the analytical solutions resulting from it are thoroughly compared to previous available analytical solutions as well as with the results of finite element simulation.

The problem is solved via the elastic-viscoelastic correspondence principle. The viscoelastic problem when transformed to the Laplace domain corresponds to an elastic problem. In the more general case of a biaxial loading such a problem has been solved previously.[10–12] One approach[10,11] was based on the use of the Airy stress functions; its generalization to the different cases of e.g. loading conditions at infinity or on the boundary of the hole is not straightforward. Another reported approach[12] is based on a Laurent series expansions of the Kolosov-Muskhelisvili potentials. Unfortunately, it seems that the authors of this last publication – possibly being unaware of previously published solutions – have not been able to compare their results, while early solutions reported in the pre-computer era could not be compared with the results of any numerical simulations.

The solution in Laplace domain presented here has a more compact form than any previously reported solutions. The comparison with all the previously published solutions is not always possible due to either insufficient information and/or misprints in some relevant publications. We therefore perform comparisons with some well documented solutions, when possible, but also with the finite element simulations. In particular, the solution in the Laplace domain (which is the solution of the corresponding elastic problem) for a bi-axial stress and a lined hole is compared to the result of the finite element elastic simulation with both a bi-axial far-field stress at infinity and a hydrostatic pressure acting on the lining.

To obtain the solution in the time domain, the Laplace inversion is done either analytically or numerically using e.g. Stehfest algorithm. Here again, to reduce the possibility of misprints, a Mathematica script is provided as a ready-to-use tool for a variety of viscoelastic models. These models reflect the experimentally observed fact[13] that rocks typically respond elastically under volumetric loading and viscoelastically in shear. While our approach is quite general, examples for five viscoelastic models in shear are presented in more details. For the case of a hydrostatic load and an incompressible rock, we compare the solutions obtained here to some previously reported solutions for particular viscoelastic models. Finally for illustration, we discuss the response of the structure in the time domain using the viscoelastic properties of a mudstone modeled as a Burgers viscoelastic material.[14]

2. Problem formulation

The mathematical model of the problem involves an isotropic, linearly viscoelastic plane containing a circular hole of radius R_2. Two different scenarios of the rock-lining interactions are considered: (i) the lining is not inserted and (ii) perfect bond is assumed between the lining and the material of the rock mass. In the first scenario, the boundary of the hole can be subjected to the pressure that varies with time as $-f(t)p_m$, where $f(t)$ is the function of time (e.g., in case of borehole excavation, it is associated with the use of drilling mud). In the second scenario, the lining is modeled by a viscoelastic ring characterized by an internal radius R_1 and external radius R_2, as shown in Fig. 1. As can be seen from the figure, the Cartesian coordinate system is chosen such that its origin coincides with the center of the circle. Let L_k, $k = 1, 2$ denote the internal (external) boundary of the ring and L_2 be the boundary of the excavated hole. In both scenarios it is assumed that the boundary L_2 of the hole (scenario i) or internal boundary L_1 of the ring (scenario ii) is subjected to the pressure $-f(t)p_m$.

The viscoelastic model that describes rock's rheology is not specified at this point, as the method enables the adaptation of various models. The evolution of stresses, strains and displacements in the rock and inside the lining are to be determined.

3. Basic equations

The problem considered here can be decomposed into two problems: one for the lining and a complementary problem for the viscoelastic plane containing a cavity. For the scenario (ii), the two problems are interrelated by the condition of perfect bond between the viscoelastic plane and the inserted lining.

The analysis is based on the use of elastic-viscoelastic correspondence principle. The system of governing equations in the Laplace domain is obtained by using the transformed Somigliana's identities, which are the corollaries of the reciprocal theorem (the principle of virtual work). The identities express the displacements or stresses at a point within an elastic region in terms of the integrals of the tractions and displacements (some of which are not known) over its boundary. The unknown boundary displacement and/or tractions can be obtained from the prescribed boundary data using the limiting process in which the inner point of the region is allowed to approach the boundary. The details of this approach are presented in subsequent sections.

3.1. Elastic-viscoelastic correspondence principle

According to the principle, the solution of the problem in the time domain can be obtained from the solution of the corresponding elastic problem by applying the inverse Laplace transform to s-dependent quantities,[15] where s- is the transform parameter.

The Laplace transform of a real function $f(t)$ is defined as

$$\hat{f}(s) = \int_0^\infty f(t)\exp(-st)dt \tag{1}$$

where s in general is a complex number. The inverse Laplace transform is given by the Bromwich integral:

$$f(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \hat{f}(s)\exp(st)ds \tag{2}$$

where γ is a vertical contour in the complex plane chosen in such a way that all singularities of $\hat{f}(s)$ are located to the left of it.

3.2. Viscoelastic models

Under a unidimensional loading, a linear viscoelastic model (see some examples on Fig. 2) can be visualized as a combination of linear springs and linear viscous dashpots whose constitutive equations are respectively

$$\begin{aligned} \sigma &= E\varepsilon \\ \sigma &= \eta\dot{\varepsilon} \end{aligned} \tag{3}$$

where E is the Young modulus of the spring and η is the viscosity of the dashpot; the dot denotes the time derivative.

The constitutive differential equation for a specific model can be obtained from Eqs. (3) combined with the expressions for the strains and/or stresses associated with that model. E.g., using the fact that the total strain in the Maxwell model (Fig. 2b) is the sum of strains in each element, one obtains with the use of Eqs. (3) the following differential equation: