International Journal of Rock Mechanics and Mining Sciences 105 (2018) 145–159

Contents lists available at ScienceDirect

International Journal of Rock Mechanics and Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

# Simple three-dimensional Mohr-Coulomb criteria for intact rocks

Hua Jiang[a,b]

[a] Key Laboratory for Bridge and Tunnel of Shaanxi Province, Chang'an University, Xi'an 710064, China
[b] School of Highway, Chang'an University, Xi'an 710064, China

ARTICLE INFO

*Keywords:*
Strength criterion
Mohr-Coulomb
Mogi-Coulomb
Rock
Polyaxial compression

ABSTRACT

A major component of the Mohr-Coulomb (M-C) criterion is that the failure of the material is independent of the intermediate principal stress $\sigma_2$ along the potential planes of sliding. To account for the effect of $\sigma_2$, a linear relationship between the octahedral shear stress $\tau_{oct}$ and the mean stress $\sigma_{m,2}$ on the plane of failure (called the Mogi-Coulomb criterion) is widely used to represent rock failure in polyaxial compression (PXC) in practice. In this research, a linear relationship between the equivalent stress $\sigma_e$ (or $\tau_{oct}$) and major principal stress $\sigma_1$ (or minor principal stress $\sigma_3$) is proposed, which has a similar expression as the Mogi-Coulomb criterion. The relationship between the proposed failure criteria and M-C criterion is like the relationship between the Tresca and Von-Mises criteria. The material parameters can be related to the M-C strength parameters, and can be easily obtained from conventional triaxial tests. The proposed criteria are numerically calibrated against triaxial compression (TXC) and PXC data sets using three different methods. Sensitivity of the range of TXC stress data employed for fitting on the prediction results in PXC are further discussed. Comparisons of two proposed criteria with the Mogi-Coulomb criterion for rocks in PXC demonstrate that one new criterion with convex failure envelope gives the best performance for seven rock types and provides an acceptable modelling for the rest of rock types. Another new criterion and the Mogi-Coulomb criterion result in large prediction error for three (or two) rock types though they can give best performance for some rock types.

## 1. Introduction

The Mohr-Coulomb (M-C) criterion is the best known failure model for geo-materials and is also the most frequently used in practice. It has a simple expression in terms of the major and minor principal stresses, $\sigma_1$ and $\sigma_3$, respectively, and has a clear physical meaning of the material parameters. A major component of this failure criterion is that the failure of the material is independent of the intermediate principal stress, $\sigma_2$, along the potential planes of sliding. Several three-dimensional (3D) or polyaxial failure criteria (such as Drucker-Prager,[1] Lade-Duncan,[2] Matsuoka-Nakai[3,4] and Yu MH[5]), which coincide with the M-C criterion in triaxial compression (TXC) or even triaxial extension (TXE) states, have been developed to consider the effect of $\sigma_2$ in stress states other than TXC. The deviatoric plane shapes of these 3D failure criteria are bounded by the M-C (lower bound) and twin-shear criteria (upper bound).[6] It is concluded in the ISRM suggested method that the best fitting curve for all polyaxial failure criteria obtained to date is that the octahedral shear stress $\tau_{oct}$ are power functions of the mean stress $\sigma_{m2}$ acting on the plane of failure.[7] Al-Ajmi and Zimmerman[8] proposed a linear relationship between $\tau_{oct}$ and $\sigma_{m2}$ (called the Mogi-Coulomb criterion or the linear Mogi criterion) to approximate power-law relationships, and then the involved material parameters can be related to the M-C strength parameters or known rock mechanical properties because the linear version shares the same compression and extension meridians with the M-C criterion.

In this research, two simple 3D M-C criteria are proposed based on two different expressions of the M-C criterion to represent rock failure in polyaxial compression (PXC), which have similar expressions as the Mogi-Coulomb criterion. The Mogi-Coulomb criterion is found to be a 3D generalization of the M-C criterion for PXC based on another expression of the M-C criterion. The proposed two failure criteria use linear relationships between the Equivalent stress $\sigma_e$ (or $\tau_{oct}$) and $\sigma_1$ (or $\sigma_3$), and coincide with the M-C criterion in TXC and TXE. The involved material parameters can be related to the two material parameters appearing in the M-C strength or Mogi-Coulomb strength parameters, and can be easily obtained from TXC test data. The characteristics of the failure surfaces on the deviarotic plane are pointed out and compared with that of the Mogi-Coulomb criterion.

Available TXC and PXC test data of fourteen different rock types[9–18] is used to calibrate the proposed failure criteria. Three different calibration methods are introduced and the sensitivity of TXC test data employed for fitting is further discussed. Comparisons of two new criteria with the Mogi-Coulomb criterion for rocks in PXC using the best-fitting parameters obtained from TXC test data are made.

*E-mail addresses:* huajiang2006@hotmail.com, hkustuga@gmail.com.

https://doi.org/10.1016/j.ijrmms.2018.01.036
Received 21 April 2017; Received in revised form 3 August 2017; Accepted 20 January 2018
1365-1609/ 

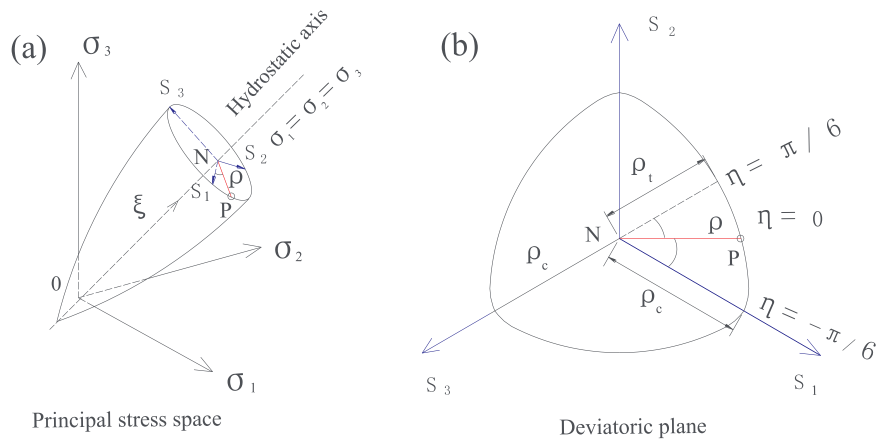


**Fig. 1.** Haigh–Westergaard space and Principal stress space.

## 2. Conventions and definitions

The failure criterion is usually expressed in terms of (and visualized in) a 3D principal stress space ($\sigma_1, \sigma_2, \sigma_3$) or a version of the 3D Haigh–Westergaard stress space ($\xi$, ρ, $\eta$).[19,20] Fig. 1 shows the relationship between the Haigh–Westergaard coordinates and principal stress coordinates, where $\xi$ is the projection on the unit vector $\bar{e} = (1, 1, 1)/\sqrt{3}$ on the hydrostatic axis, the radial distance from the failure point P to the hydrostatic axis N is given by ρ, and the Lode angle $\eta$(-π/6 ≤ $\eta$ ≤ π/6) is the measure of a rotation from axis $x$ on the deviatoric plane, which is orthogonal to (1,1,1) or the hydrostatic axis.

$$\xi = |\overrightarrow{\mathrm{ON}}| = I_1/\sqrt{3} = \sigma_{ii}/\sqrt{3} = \sqrt{3}\sigma_m \tag{1}$$

$$\rho = |\overrightarrow{\mathrm{NP}}| = \sqrt{2J_2} = \sqrt{S_{ij}S_{ij}} = \sqrt{S_1^2 + S_2^2 + S_3^2} \tag{2}$$

$$\eta = -\frac{1}{3}\arcsin\left(1.5\sqrt{3}\frac{J_3}{\sqrt{J_2^3}}\right) \tag{3}$$

where $\sigma_m$ and $I_1$ are the hydrostatic stress and the first invariant of the stress tensor $\sigma_{ij}$, respectively; $J_2$ and $J_3 = S_{ij}S_{jk}S_{ki}/3$ are and the second and third invariants of the deviator stress tensor $S_{ij}$, respectively; the deviator stress tensor $S_{ij}$ can be linked to the stress tensor $\sigma_{ij}$ by $S_{ij} = \sigma_{ij} - \sigma_{ii}\,\delta_{ij}/3$ and ($S_1, S_2, S_3$) are the principal values of $S_{ij}$.

The interchange relationships between the principal stresses, and the stress invariants are given by[19,20]

$$\sigma_1 = \sigma_m + \frac{2}{\sqrt{3}}\sqrt{J_2}\,\sin\left(\eta + \frac{2\pi}{3}\right) \tag{4}$$

$$\sigma_2 = \sigma_m + \frac{2}{\sqrt{3}}\sqrt{J_2}\,\sin\eta \tag{5}$$

$$\sigma_3 = \sigma_m + \frac{2}{\sqrt{3}}\sqrt{J_2}\,\sin\left(\eta - \frac{2\pi}{3}\right) \tag{6}$$

In TXC ($\sigma_1 \geq \sigma_2 = \sigma_3$), the stress state lies on the compressive meridian ($\eta$ = -π/6) and in TXE ($\sigma_1 = \sigma_2 \geq \sigma_3$), the stress state lies on the extension meridian ($\eta$ = π/6).

## 3. Simple 3D M-C criteria

### 3.1. Different expressions of M-C criterion

Let us consider compression stresses as positive and tensile stresses as negative quantities, which is commonly used for rock materials. Moreover, it is assumed that the three principal stresses are ordered according to $\sigma_1 \geq \sigma_2 \geq \sigma_3$.

The failure function of the two-parameter M-C criterion can be expressed in terms of the principal stresses as[21,22]

$$\sigma_1 - \sigma_3 = 2c\cos\varphi + (\sigma_1 + \sigma_3)\sin\varphi \tag{7}$$

Or

$$\sigma_3 = \frac{1 - \sin\varphi}{1 + \sin\varphi}\sigma_1 - \frac{2c\cos\varphi}{1 + \sin\varphi} = \frac{\sigma_t}{\sigma_c}\sigma_1 - \sigma_t \tag{8}$$

Or also as

$$\sigma_1 = \frac{2c\cos\varphi}{1 - \sin\varphi} + \frac{1 + \sin\varphi}{1 - \sin\varphi}\sigma_3 = \sigma_c + \frac{\sigma_c}{\sigma_t}\sigma_3 \tag{9}$$

where $c$ and $\varphi$ are two material properties termed the cohesion strength and internal friction angle of the material, respectively (see Fig. 2). $\sigma_c$, $\sigma_t$ are the theoretical uniaxial compression and uniaxial tension strength (a hypothetical value), and can be related to the cohesion strength and internal friction angle by following expressions:

$$\sigma_c = \frac{2c\cos\varphi}{1 - \sin\varphi} = 2c\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \tag{10}$$

$$\sigma_t = \frac{2c\cos\varphi}{1 + \sin\varphi} = 2c\tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) = 2c\cot\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = \frac{4c^2}{\sigma_c} \tag{11}$$

According to the M-C criterion, the normal and shear stresses ($\sigma_n$, $\tau_{oct}$) on the failure planes with normal inclined to $\sigma_1$ at an angle $\pm\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$ as shown in Fig. 2.

Based on Eq. (8), the M-C criterion can be written as

$$\sigma_1 - \sigma_3 = \sigma_t + \left(1 - \frac{\sigma_t}{\sigma_c}\right)\sigma_1 \tag{12}$$

which will be used to construct the proposed 3D M-C Criterion A.

Based on Eq. (9), the M-C criterion can be also expressed as

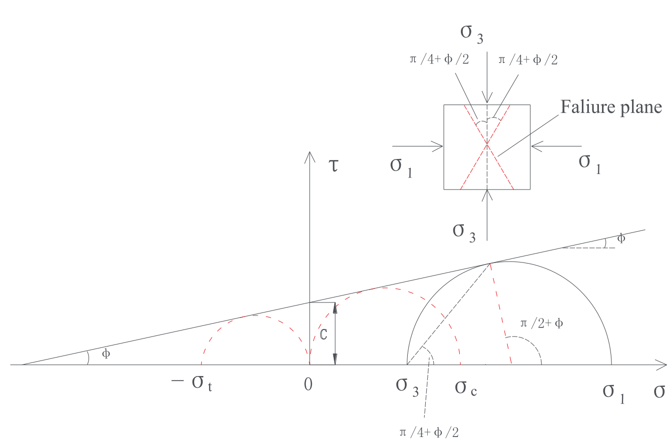


**Fig. 2.** Mohr–Coulomb failure criterion.