International Journal of Rock Mechanics and Mining Sciences 103 (2018) 107–115

Contents lists available at ScienceDirect

International Journal of Rock Mechanics and Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

A Modified Cubic Law for single-phase saturated laminar flow in rough rock fractures

Zhihe Wang*, Chaoshui Xu, Peter Dowd

School of Civil, Environmental and Mining Engineering, The University of Adelaide, Adelaide 5005, Australia

ARTICLE INFO

Keywords:
Modified Cubic Law
Rough rock fractures
Navier-Stokes Equations
Aperture field modification

ABSTRACT

The purpose of the study presented was to develop a Modified Cubic Law (MCL) for single-phase saturated laminar flow in rough rock fractures. Based on the fundamental assumptions made for the Cubic Law (CL), the proposed MCL incorporates modifications to the aperture field by considering flow tortuosity, aperture variation and local roughness effects. We assess the performance of the MCL by applying it to synthetic fractures with different surface roughness and varying aperture and compare the outputs to numerical simulation results from solving the Navier-Stokes Equations and results from previous versions of CL-based models. In general, the MCL performs well in predicting the volumetric flow rate in synthetic fractures with deviations (*D*) from simulation results ranging from −11.1% to 8.4% and an average effective deviation of 4.7%. The proposed model retains the simplicity of CL models and improves the accuracy of flow prediction in terms of single-phase saturated laminar flow in rough rock fractures and it can be extended to analyse other hydro-related problems.

1. Introduction

Rock masses commonly include fractures or faults resulting from either tectonic activities or human disturbances. These fractures, at different scales, often act as active conduits for fluid flow (e.g., water). A sound knowledge of the controlling mechanism of fluid flow in rock fractures is of interest in various research fields and engineering applications including groundwater flow, solute transport, enhanced geothermal systems and hazardous waste disposal [e.g.,[1–3]]. Modelling the complex behaviour of flows in fractured rock masses requires a fundamental understanding of the hydraulic behaviour of discrete single fractures.

The Navier-Stokes Equations (NSE) are widely accepted as the governing equations for steady-state incompressible flow in rock fractures.[4,5] By considering mass conservation, they can be written as:

$$\rho(\boldsymbol{u}\bullet\nabla)\boldsymbol{u} = -\nabla P + \mu\nabla^2\boldsymbol{u} \tag{1}$$

$$\nabla\bullet\boldsymbol{u} = 0 \tag{2}$$

where ρ is the fluid density, μ is the dynamic viscosity, $\boldsymbol{u}$ is the velocity and P is the pressure. Although in theory the NSE provide an exact description of three-dimensional flow in rock fractures, the computational cost required to solve three-dimensional partial differential equations is prohibitive for applications beyond the microscopic scale.[6]

The common approach to modelling fluid flow through rock fractures is to assume that a fracture consists of two smooth parallel plates separated by a constant aperture.[7] Under this assumption, the Cubic Law (CL) can be derived[8] and has the following form:

$$Q = -\frac{W\rho g b^3}{12\mu}\frac{\Delta H}{L} \tag{3}$$

where Q is the volumetric flow rate, W is the fracture width, g is the gravitational acceleration, b is the fracture aperture, H is the hydraulic head and L is the fracture length.

The CL is widely used for flow prediction in rock fractures in many fields due to its simplicity. However, real fractures are often formed by two surfaces with anisotropic roughness and varying aperture, which lead to a three-dimensional non-uniform tortuous flow field rather than the one-dimensional Poiseuille flow assumed by the CL.[6,9] Numerous laboratory experiments indicate that the CL may produce significant errors in the prediction of flow through rock fractures.[10,11] Another approach is to account for the spatial variability of the fracture aperture by assuming that a particular CL applies at each explicit location. This is known as the Local Cubic Law (LCL) and was derived from the NSE using lubrication theory[12]; it is also called the Reynolds equation and takes the form:

$$\nabla\bullet\left[\frac{\rho g b^3}{12\mu}\nabla H\right] = 0 \tag{4}$$

The validity of LCL has been questioned in previous studies. It

* Corresponding author.
E-mail address: zhihe.wang@adelaide.edu.au (Z. Wang).

https://doi.org/10.1016/j.ijrmms.2017.12.002
Received 2 March 2017; Received in revised form 18 December 2017; Accepted 31 December 2017

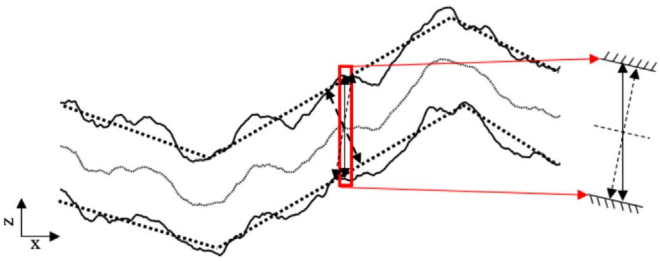

Fig. 1. Different definitions of fracture aperture with solid arrows representing the vertical aperture, dotted arrows for the aperture perpendicular to the local cell centreline[13] and the bold dashed arrow for the segment aperture.[16] The bold dashed lines are the segment walls and the dashed lines refer to the centrelines of the local cells. One typical local cell is illustrated as the area marked by the block.

assumes a flat plane for the fracture mid-surface, whereas rough fractures are more likely to have a tortuous mid-surface,[13,14] which is incompatible with the assumption of a parabolic velocity profile and results in an over-estimation of the flow rate by a factor of at least 1.75.[15]

To avoid the computational cost of solving the NSE and to improve the performance of the over-simplified CL and LCL, research has been focused on developing alternative models based on modifying the CL and the LCL. Statistical parameters (e.g., relative roughness[17]; surface roughness[18]) and empirical parameters (e.g., joint roughness coefficient[19] (*JRC*)) are incorporated to modify the CL by using data from flow experiments or values generated by the LCL to derive a relationship between the vertical aperture (also known as apparent aperture) and the hydraulic aperture. As these models ignore flow behaviour at the local scale, their performance varies significantly with the fracture void geometry. Ge[13] derived an alternative flow governing equation based on aperture field modification, in which the aperture at each location is modified by considering the geometric properties of every local cell (see Fig. 1). This approach is supported by Konzuk and Kueper,[15] who evaluated the modified aperture field with CL models and found that the CL calculated with either the geometric mean aperture or incorporating surface roughness factors predicted the flow rate within ± 10% of the observed values for Reynolds number less than 1.0. Wang et al.[14] developed an improved CL-based model by extending Ge's approach to account for the roughness effect of local cells as a means of aperture field modification. However, the validity of modifying the aperture field in terms of the geometric properties of each local cell may depend on the resolution of the aperture field data and the variation of both aperture and surface roughness.[16,20,21] Oron and Berkowiz[16] conducted a leading-order approximation to two-dimensional Navier-Stokes Equations and concluded that the cubic law assumptions may be valid within certain segments as long as both the non-dimensional local roughness parameter and the aspect ratio of the segments are significantly less than 1.0 (i.e., both walls are relatively smooth and the segment length is much longer than the segment half-aperture). However, in the absence of non-segmented area treatment and quantification of the local roughness effect, its application to fracture flow prediction is limited.

The purpose in this work is to present a Modified Cubic Law (MCL) with improved performance in terms of predicting the volumetric flow rate in rough rock fractures. The aperture field is modified by considering the effects of flow tortuosity, aperture variation and local roughness. Numerical simulations of fracture flow were conducted for synthetic fractures with varying surface roughness and aperture by solving the NSE to assess the validity of the proposed MCL. Previous models are also used for comparisons to further test the robustness of the MCL presented here.

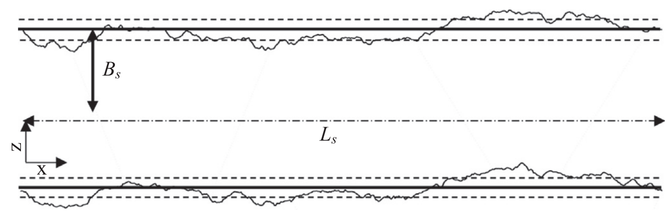

Fig. 2. Illustration of one segment with B_s the half-aperture and L_s the segment length.[16]

2. Model development

2.1. Assumptions for flow direction in segments

We began by making assumptions for the flow direction field of a three-dimensional fracture with surface roughness. Take a cross-section of a fracture segment, as shown in Fig. 2; B_s is the average half-aperture and L_s is the length of this fracture segment. It can be assumed that Poiseuille flow holds in the segment and the flow direction is governed by the geometry of the segment when B_s is much smaller than L_s and the roughness of both walls along the segment is limited.[16] These two conditions require:

$$\delta = \frac{B_s}{L_s} \ll 1 \tag{5}$$

$$\varepsilon = \max\left(\frac{\sigma_u}{B_s}, \frac{\sigma_l}{B_s}\right) \ll 1 \tag{6}$$

where δ is the aspect ratio and ε is the non-dimensional local roughness parameter; σ_u and σ_l are the standard deviations of the roughness variations of upper and lower walls, respectively. Under both conditions, the representative flow direction follows the orientation of the geometric centreline of the segment.[13] The segments of each fracture can be determined by calculating δ and ε along each fracture cross-section using their assigned maximum values. We discuss the range and determination of δ and ε in Section 4.1.

2.2. Consideration of flow tortuosity

At the scale of a single fracture, flow tortuosity is defined as the ratio of the three-dimensional flow path length to the straight-line distance of fracture length.[22] In this study, the tortuosity τ of each segment is defined as the ratio of flow path distance d_f to the straight-line distance d_s projected in the fracture plane:

$$\tau = d_f/d_s \tag{7}$$

The vertical aperture at each location within the approximated segment with walls of averaged straight lines (see Fig. 2) can be determined according to the geometry of the approximated segment. As the aperture, defined in the CL, should be perpendicular to the flow direction, vertical apertures need to be translated into flow-oriented apertures defined by the segment aperture b_{sf}. We use the formula proposed by Ge[13] to obtain the relation between the vertical aperture and b_{sf} within the segment:

$$b_{sf} = b_v \frac{2\cos\alpha_u \cos\alpha_l}{\cos\frac{\alpha_u - \alpha_l}{2}\ (\cos\alpha_u + \cos\alpha_l)} = b_v \bullet F_t \tag{8}$$

where b_v is the vertical aperture, α_u and α_l are the inclination angles of the upper and lower walls respectively. Connective transmissivity[6,23] (T) between each local cell within the segment can be approximated using the harmonic mean of adjacent apertures:

$$T = \frac{2b(i)^3 b(i+1)^3}{b(i)^3 + b(i+1)^3} \tag{9}$$