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DEM analysis of failure mechanisms in the intact Brazilian test

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A B S T R A C T

A comprehensive numerical study is conducted to examine the failure behaviors in the intact Brazilian test. Brazilian test is considered a robust laboratory test to obtain an indirect measure of the uniaxial tensile strength of quasi-brittle materials such as rocks. Though validity of the Brazilian test is based on the premise that the diametrical splitting failure is caused by tensile crack growth from the center of the specimen (Scenario I), an alternative indentation-type of failure mechanism (Scenario II), where the splitting failure pattern forms as a result of cracks emanating from the crushed zones adjacent to the loading areas, is often observed experimentally. In this work, how the failure mechanisms and consequently the Brazilian tensile strength are affected by the material properties and the sample size are investigated using the DEM code PFC2D/3D. A novel displacement-softening contact model is implemented in PFC2D/3D so that materials with realistic uniaxial compressive over tensile strength ratios as high as ~ 30 could be modeled. Formulation of the softening contact model and the effect of the micro-scale softening parameter on the macro-scale mechanical properties are first described. Intact Brazilian test is modeled in both 2D and 3D. We show that the failure scenarios transition from Scenario I to Scenario II, if the compressive over tensile strength ratio decreases or the sample size increases. For a low strength ratio material, if the failure mechanism follows Scenario II, the nominal Brazilian tensile strength could potentially underestimate the intrinsic tensile strength. Implications of this numerical analysis to laboratory testing and to the calibration of material properties in DEM modeling in general are also discussed.

1. Introduction

Uniaxial tensile strength of quasi-brittle materials such as rocks and concretes is often measured indirectly by the Brazilian test. In the test, a circular disk-shaped specimen under diametral compression is loaded to fail in a splitting pattern. Essential to the justification for the Brazilian test is that the elastic solution[1] predicts that the tensile stress perpendicular to the loading axis is nearly constant in a large portion of the central loading plane. Meanwhile, experimental observations[2–6] suggest that failure in confined extension still occurs according to the uniaxial tensile strength, if the magnitude of the compressive principal stress in the biaxial stress state is no more than a few times of the tensile stress. The Brazilian tensile strength (BTS) is therefore considered a good measure of the uniaxial tensile strength (UTS) with the premise that tensile failure initiates from the center of the specimen and propagates unstably towards the two loading platens. Given the specimen diameter D and thickness t, the nominal Brazilian strength σ_B based on the peak load P can be written as,

$$\sigma_B = \frac{2P}{\pi D t} \tag{1}$$

Compared to the direct tension test, the Brazilian test is relatively robust to perform. Nevertheless, since the location of crack initiation is difficult to be unambiguously discerned from visual inspection after routine testing, validity of the results has often been questioned (see review in 7–9). Much of the discussion in the earlier literature focuses on how the loading platens and the contact condition/area affect the failure mechanisms. As pointed out by Fairhurst [10] if the angle of the loading contact areas is small and/or the material has a relatively low compressive over tensile strength ratio, failure could occur away from the center and the tensile strength could be underestimated. In other words, there is an alternative indentation-type of failure scenario where the splitting failure pattern forms as a result of cracks emanating from the crushed zones beneath the loading areas.

Existence of both failure scenarios has been verified through a variety of experimental techniques, e.g., high speed photography of the photoelastic patterns,[11] loading/unloading process controlled by lateral displacement,[12,13] strain gages,[14] acoustic emission[15] and digital image correlation.[16] Similar evidences were also obtained based on the images from a high-speed camera in the dynamic Brazilian tests.[17] These experimental findings were corroborated by numerical analyses,[18,9] though we should recognize that the failure mechanisms in numerical

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analyses are largely dictated by the constitutive models assumed *a priori*.

The difference between BTS and UTS obtained from laboratory experiments is well recognized. Depending on the rock types, BTS has been found to both overestimate and underestimate the UTS. The ratio, σ_B/σ_t, was found to scatter in between 0.56 for trachyte and 1.86 for sandstone[19]. Rock type specific correlations were suggested[19]: $\sigma_t = f\sigma_B$, $f = 0.9$ for metamorphic, $f = 0.8$ for igneous and $f = 0.7$ for sedimentary rocks.

Meanwhile, in some studies,[20,15] Brazilian test has been reported to exhibit size effect, namely, the dependence of the Brazilian tensile strength on the test sample size. But the size effect is concluded statistically insignificant in others.[13,21] Using perhaps the largest Brazilian test sizes, $D \in [0.1, 3]$ m and $t = 0.5$ m, Hasegawa et al.[22] showed that BTS of concrete decreases with the diameter, but becomes nearly constant when the diameter is large enough. However, a possible weak reversal of the decreasing trend was also suggested,[23] namely, as the diameter becomes large, BTS seems to increase instead of decreasing before reaching the asymptote. From a theoretical point of view, neither Weibull's theory[24] of statistical distribution of weakness in the material nor Bazant's size effect argument based on stress redistribution and growth of the process zones near the crack tips for geometrically similar samples[25–27] could explain the asymptote in BTS as the sample size becomes large.

Over the last few decades, discrete element method (DEM)[28] has become an indispensable numerical tool to model progressive failure in quasi-brittle materials. A prerequisite in DEM modeling is to calibrate the material properties by identifying a set of micro-scale parameters for the particles and the contacts so as to yield desirable properties at the macro-scale. We notice in the literature that in DEM modeling with a randomly packed spherical particle assembly, while BTS is often reported as the tensile strength for the purpose of material properties calibration, to the best of our knowledge, there is no indication that the failure mechanism in those studies is indeed in form of tensile crack initiation from the center.

In order to better understand how the failure mechanisms and the sample size in the Brazilian test affect the BTS and how BTS compares with UTS, we conduct a comprehensive numerical study using the DEM code PFC2D/3D[29,30] to model the intact Brazilian test. In this work, we focus on the failure mechanisms and their dependence on the mechanical properties and the sample size. The connection between the failure mechanisms and the BTS, the size effect in BTS and the comparison between BTS and UTS will be reported in a separate study. A novel aspect of the present work is that a new contact model accounting for displacement-softening in the contact bond strength is implemented in PFC2D/3D. As such, not only realistic rock properties, especially the uniaxial compressive over tensile strength ratio, can be obtained, failure behaviors as consistent with experimental evidences from the Brazilian test, can also be reproduced in the numerical simulations. In particular, diametrical splitting failure resulted from nucleation of a center crack can now be simulated numerically in a randomly packed spherical particle assembly.

Formulation of the softening contact model and the effect of the micro-scale softening parameter on the macro-scale properties are first described. Intact Brazilian test is then modeled in both 2D and 3D to explore how the failure scenarios are affected by the material properties as well as the sample size. Implications of this numerical analysis to laboratory testing and to the calibration of mechanical properties in DEM modeling in general are also discussed.

2. Numerical model setup

2.1. Displacement-softening contact model

Implementation of the displacement-softening contact model in PFC2D/3D is realized by incorporating a softening force-displacement relationship into the default parallel bond model option. A parallel bond in PFC can be envisioned as a series of elastic springs distributed over the contact area between a pair of particles. In addition to the normal and shear contact forces, bending moments can be transmitted through the contact between particles.

The default parallel bond contact model[29,30] has two components and can be described by two groups of micro-scale parameters: 1) particle-particle contact (point contact): normal and shear stiffnesses, K_n and K_s ([F/L]), and friction coefficient μ; 2) the parallel bond (area contact): apparent normal and shear stiffnesses, $\overline{k}_n$ and $\overline{k}_s$ ([F/L^3]), the normal and shear bond strengths, $\overline{\sigma}_c$ and $\overline{\tau}_c$ (F/L^2), and the parallel bond radius multiplier, $\overline{\lambda}$. The radius multiplier $\overline{\lambda}$ defines the contact area radius for the parallel-bond via $\overline{R} = \overline{\lambda}\min(R_A, R_B)$, where R_A and R_B are the radii of the two particles in contact. Since the stiffnesses for the two types of contact, which act in parallel, differ in dimensions, it is more convenient to specify apparent moduli as the input parameters instead. For example, the normal stiffnesses K_n for the point contact and $\overline{k}_n$ for the parallel bond can be determined from,

$$K_n = \frac{4R_A R_B}{R_A + R_B}E_c \quad \overline{k}_n = \frac{\overline{E}_c}{R_A + R_B} \tag{2}$$

where E_c and $\overline{E}_c$ are the apparent moduli for the point contact and the parallel bond, respectively. The point contact is linearly elastic and frictional, where the normal and shear forces exist only if the two particles have an overlap.

For the area contact or the parallel bond, before reaching the softening condition, the contact forces, $\overline{F}_n$ and $\overline{F}_s$, and the bending moment, $\overline{M}$, follow the linear relationships with the parallel bond stretch $\overline{\delta}_n$, slip $\overline{\delta}_s$ and the angle of relative particle rotation $\overline{\theta}$,

$$\overline{F}_n = -\overline{k}_n A\overline{\delta}_n \quad \Delta\overline{F}_s = -\overline{k}_s A\Delta\overline{\delta}_s \quad \Delta\overline{M} = -\overline{k}_n I\Delta\overline{\theta} \tag{3}$$

where A and I are the cross sectional area and the moment of inertia of the parallel bond, respectively. Note that compression is assumed positive and $\overline{\delta}_n > 0$ denotes the bond elongation along the contact axis.

The softening force-displacement relationship is incorporated into the normal component of the contact (see Fig. 1). Softening occurs when the normal contact force $\overline{F}_n$ in a parallel bond reaches a limit defined by the normal bond strength $\overline{\sigma}_c$, i.e.,

$$\overline{F}_{n\,\max} = -\overline{\sigma}_c A \tag{4}$$

The force-displacement relationship during the softening stage can be expressed as,

$$\overline{\delta}_n = \frac{\overline{\sigma}_c}{\overline{k}_n} + \frac{\overline{\sigma}_c + \overline{F}_n/A}{\beta\overline{k}_n} \tag{5}$$

where the softening coefficient β defines the ratio between the loading and softening stiffnesses, i.e., $\beta = \overline{k}_u/\overline{k}_\ell$ and $\overline{k}_\ell = \overline{k}_n A$. The perfectly brittle case when $\beta \to \infty$ is the default parallel bond option in PFC and

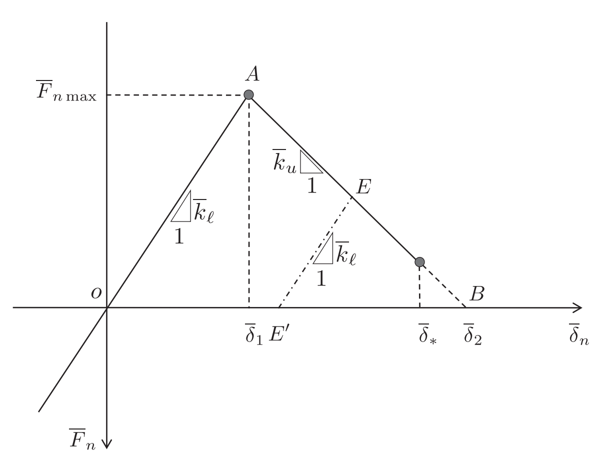

Fig. 1. Normal force-displacement contact law for the parallel bond.