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Closed-form elastic solution for irregular frozen wall of inclined shaft considering the interaction with ground

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ABSTRACT

An analytical solution is derived that provides a closed-form formulation for stresses and displacements of a deep inclined shaft with irregular frozen wall liner in an infinite elastic medium, subjected to a non-uniform stresses. This solution is based on: (i) the conformal mapping of a doubly connected domain of prescribed shape onto a circular ring by an appropriate numerical optimization scheme which is achieved by MATLAB program, (j) the complex variable method. For a particular case, the distributions of hoop stress and radial displacement inside frozen wall are presented with analytical solution. The accuracy of stress functions is verified by comparison of different number of coefficients in stress functions and it show that the number of coefficients $k \geq 15$ is advised. The validity of analytical solution is verified by a comparison between its result and that obtained from the finite element program ANSYS. Noting that the shaft excavation can be regarded as relaxation of initial ground stresses around shaft excavation boundary, this paper presents the mechanical problem as two separate problems: (i) initial stresses and displacements model where stresses keep constant and displacements equal zero around frozen wall and ground, (j) stresses relaxation model with relaxed normal and shear stresses acted on excavated boundary. The solution is worked out by the principle of superposition in elastic medium. The Young's modulus ratio (β) of the frozen wall to the ground reflects the influence of ground to frozen wall, and the influence decreases with the growth of β. The proposed solution can provide analytical basis for frozen wall design and be used as a quick-solver for back-analysis of in situ stresses and displacements.

1. Introduction

Artificial ground freezing (AGF) is a special construction method, which is used to freeze the soft or water-bearing ground into an enclosed structure, to increase the strength and stability of ground soil or rocks, and to make them impervious to water seepage.[1] AGF has been widely used in mine construction, urban underground tunnel, and deep foundation excavation and so on.

Tunnel and shaft usually have a circular cross section because of its structural advantages due to the large surrounding pressures. Analytical solutions for excavation of circular holes in elastic and homogeneous materials subjected to uniform stresses or non-uniform stresses are discussed in textbooks of elastic theory and rock mechanics due to their importance as basic tools for introducing the subject of underground tunnel excavations.[2,3] Einstein and Schwartz[4] perhaps at the first time worked out the solution for excavation and support problem of deep circular lined tunnel in an infinite elastic medium subjected to non-uniform stresses. However, they ignored the effect of support delaying. As supplementary, with consideration of the delaying effect of support and different contact conditions on the interface between ground and lining, Carranza-Torres et al.[5] found an analytical elastic solution for deep circular lined tunnel. Mason and Stacey[6] considered three contact conditions on the interface between ground and support, and then analyzed the problem of concrete spraying support in deep circular tunnel subjected to a uniform shear stress at infinity.

In most cases of tunnel and inclined shaft excavation, non-uniform stresses are acted on models. As an effective method, complex variable method can be used to solve the problem of circular or irregular tunnels or shaft subjected to uniform or non-uniform stresses. Poulos and Davis[7] analyzed the tunnels with and without lining in his work. Pender[8] and Sagaseta[9] presented the stress-displacement solution for circular lined tunnel without regard to gravity as well. Li and Wang[10,11] achieved the solutions for deep circular lined tunnel subjected to uniform internal pressures, and stress release of excavation to support and water seepage were taken into consideration. Based on these solutions, Yang and Wang[12] obtained the solutions for circular shaft frozen wall and ground subjected to non-uniform stresses, and they considered the stresses relaxation model of shaft excavation.

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Although the circular cross section is the first choice for inclined shaft excavation, the horseshoe-like section (a section with semi-circular arch roof and straight side wall and inverted arch floor) is the most common section of inclined shaft limited to the excavation condition and complex ground condition. It is assumed that an irregular boundary in the z-plane can be mapped conformally to a circle in the ζ-plane.[13,14] The exact analytical solution for circular lined tunnel and simple irregular tunnel, like lined oval tunnel, could be worked out by complex variable theory, while only closed-form solution could be obtained for relatively complex tunnel. Based on this theory, Gercek[15] presented the solution for stresses around tunnel with conventional shapes for the first time. However, Gercek did not compute the displacement solution for the ground, and did not consider the support either. Exadaktylos et al.[16,17] presented the closed-form solutions for stresses and displacements around semi-circular and notched circular tunnels. Exadaktylos also did not take support into consideration. Huo and Bobet[18] analyzed the regularities of stresses distribution around deep lined tunnel in earthquake region subjected to shear stresses.

Clearly, all published works present solutions for circular supported tunnel or irregular tunnel without support. Different with them, frozen wall is a kind of doubly connected domain, which plays a significant role in the construction of shaft or tunnel, however there is no published work presenting the analytical solution for deep lined irregular tunnel or shaft subjected to non-uniform stresses (frozen wall is a kind of temporary liner). Now because of the irregularity of inclined shaft frozen wall, the design of inclined shaft frozen wall is based on method of circular shaft and empirical method, there is no theoretical solution for irregular frozen wall. Unfortunately, many inclined shafts were flooded because of weak frozen wall in western China. Therefore addressing the deep irregular inclined frozen shaft, this paper presents a closed-form solution of stresses and displacements for irregular frozen wall and ground with consideration of the interaction between frozen wall and ground.

2. Problem statement

This paper refers to the excavation problem of an irregular inclined frozen shaft in an infinite elastic medium under plane strain condition, subjected to non-uniform stresses, the cross section of inclined frozen shaft is shown in Fig. 1a, which consists of frozen wall and ground. In Fig. 1a, r_0 and r_1 are inner and outer radii of semi-circular arch of frozen wall, respectively, h_0 and h_1 are inner and outer heights of straight side wall, respectively, r_{d0} and r_{d1} are inner and outer radii of inverted arch, respectively.

This paper is conducted based on three assumptions presented as follows:

(1) Frozen wall and ground are regarded as elastic and homogeneous materials.
(2) Frozen wall and ground are assumed to be fully contacted, that is to say, the radial stress and shear stress are continuous at the interface between frozen wall and ground, as well as the radial and tangential displacements.
(3) The variation of body forces are ignored, as well as the frost heave, i.e., field quantities (stresses and displacements) remain unchanged after freezing.

2.1. Stresses relaxation model

The excavation of a deep inclined frozen shaft can be regarded as stresses relaxation around excavation boundary in an infinite elastic medium. Considering the stress path of shaft excavation, the problem of this paper can be divided into two new problems, stresses relaxation model (see Fig. 1a) and initial model, and the final solution is worked out by the principle of superposition in the elasticity. stresses relaxation model shows the excavation problem of an irregular inclined frozen shaft in the elastic material subjected to normal stress σ_{r0}^F and shear stress $\tau_{r\theta0}^F$ at the inner boundary, and normal displacements $u_x=0$ and $u_y=0$ at the far-boundary($r\to\infty$). Initial model shows the initial stresses and displacements of frozen wall and ground where stresses keep constant and $u_x=0$ and $u_y=0$. Fig. 1a also can be separated by two parts, mechanical model for frozen wall as shown in Fig. 1b and mechanical model for ground as shown in Fig. 1c. Fig. 1b shows the frozen wall problem with normal stress and shear stress at inner and outer boundaries. Fig. 1c shows the ground problem with normal stress and shear stress at inner boundary, and normal displacements $u_x=0$ and $u_y=0$ at the far-boundary ($r\to\infty$).

In Fig. 1, σ_{r0}^F and $\tau_{r\theta0}^F$ are radial and shear stresses on inner boundary of frozen wall, respectively. σ_{r1}^F and $\tau_{r\theta1}^F$ are radial and shear stresses on the interface of frozen wall and ground, respectively.

In this paper, complex variable method is used to work out the solution for irregular frozen wall and ground in an infinite elastic material. In the complex variable method,[19–21] the solution is expressed in terms of two analytical functions $\varphi(z)$ and $\psi(z)$, the stresses are related to these functions by the following equations

$$\begin{cases}\sigma_y+\sigma_x=4\,\mathrm{Re}\,\varphi'(z)\\ \sigma_y-\sigma_x+2\mathrm{i}\tau_{xy}=2[\bar{z}\varphi''(z)+\psi'(z)]\end{cases} \tag{1}$$

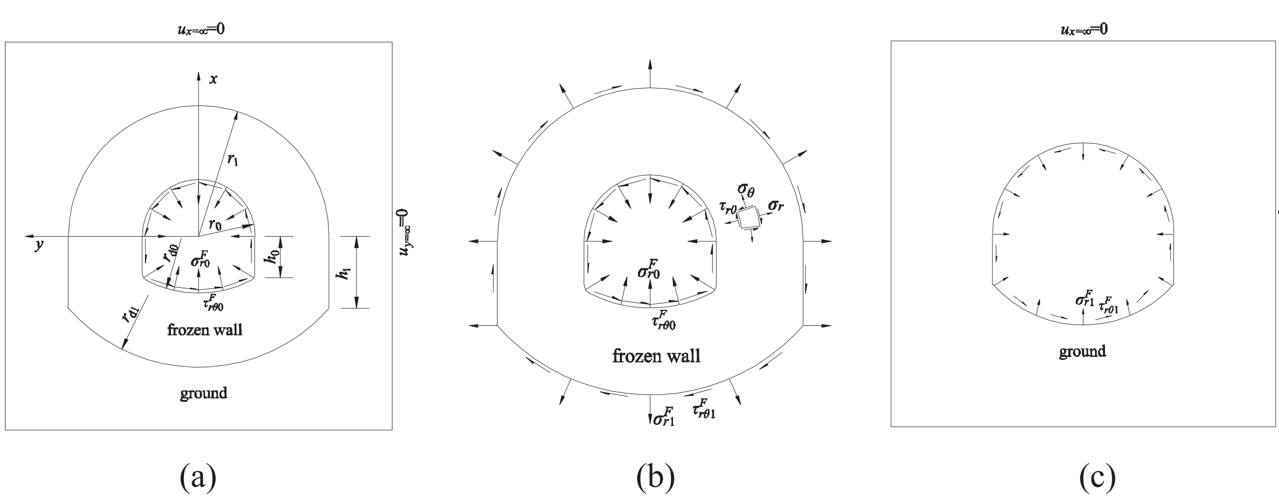

Fig. 1. Mechanical model for (a) stresses relaxation considering the interaction of frozen wall and ground with relaxation stresses on inner boundary of frozen wall in an infinite plane which can be separated into two parts: (b) frozen wall model and (c) ground model.