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Limit analysis of failure mechanism of tunnel roof collapse considering variable detaching velocity along yield surface

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1. Introduction

In spite of much progress in the advancement of theories and technologies of geotechnical engineering, the roof collapse in deep tunnels remains a great threat to life and property especially when excavation proceeds in weak rock mass. As a classical topic, the stability problems of tunnel roof have been discussed in literature, and several approaches are proposed and applied efficiently. In early period, the pressure arch hypothesis was put forward to estimate the stability of underground cavity, including tunnel, coal mine, chamber and so on.[1,2] Based on the linear Mohr-Coulomb failure criterion, those semi-empirical and semi-theoretical methods are extremely simple and concise, meanwhile, with sufficient precision in practical projects where only a general requirement is demanded. However, with the rapid development of national infrastructure construction, tunnel excavation is increasingly exposed to complex conditions where the mechanical properties of the rock mass in situ show a strong nonlinearity.[3,4] Especially when the tunnel is surrounded by weak rock mass, a slight disturbance may cause a large deformation. To overcome this problem, a multitude of scholars spare no effort to find out more generalized methods to estimate the potential risk of tunnel roof collapse with rigorous theoretical foundations and scientific computing means.

In recent years the relevant approaches, such as limit analysis, limit equilibrium, finite element and experimental method, have been rapidly developed and widely applied in engineering to assess the stability of tunnel roof or face. By contrast, the limit analysis method has received close and extensive attentions for its tight logic and rigorous physics meaning. Mollon *et al.*[5] provided an approach to the critical retaining pressure applied on tunnel face by exhibiting a three-dimensional multi-block slip failure mechanism. To further verify its correctness, the three-dimensional numerical simulation was performed as well via the finite different commercial software. Wang *et al.*[6] established a two-dimensional multi-block slip failure mode according to limit analysis method upon which the stability of tunnel roof and surrounding rock were investigated simultaneously. However, the main trouble of those studies is that the failure boundary of tunnel roof or face must follow the assumed curve. And it is only allowed to vary within a certain range which is usually achieved by constraint optimization. As a result, the unconventional shape of failure domain may be neglected due to the preconceived viewpoint.

Unlike those methods, Fraldi and Guarracino[7–9] investigated the deep tunnels in the framework of plasticity theory and derived the analytic solutions of the velocity discontinuities along yield surface in tunnel roof with the help of variational principle,[10] which in fact outlined the boundary of failure domain. Consequently the shape of impending block in tunnel roof was identified by establishing the equilibrium equation of the rate of energy dissipation and the rate of external work in any kinematically admissible velocity field. Employing this method, the shape of collapsing block is not necessarily assumed in advance, but rather purely deduced using the upper bound theorem and analytical mathematical technique. In terms of the material nonlinearity, the Hoek-Brown failure criterion is introduced instead of the Mohr-Coulomb yield rule. It is worth noting that despite its validity which has been verified by many scholars, this approach can be further improved. By examining the method in detail, the authors find that it seems unreasonable to assume that the velocity direction of the particle on yield surface coincides with that of the collapsing block with reference to the associated flow rule.[11,12]

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As a result, this research is specifically performed based on the failure mechanism of tunnel roof proposed by Fraldi and Guarracino,[7] and then the original approach is investigated and modified considering variable detaching velocity along yield surface. As a matter of fact, a huge trouble resides in tackling this issue because it is involved with an extremely complex variational procedure especially when taking into account of the variable velocity direction. In this paper, by introducing a simplification technology, a modified approach is proposed to discuss the failure mechanism of tunnel roof aiming to reduce the deviation as much as possible.

2. Generalized Hoek-Brown failure criterion

The Hoek-Brown failure criterion was first proposed in 1980, and exhibited the nonlinear relationship between principal stresses when the rock was damaged.[13,14] Since then, several significant changes have been made to satisfy the requirements of different users in reference to intact rock, jointed rock mass or even very poor quality rock mass. The generalized Hoek-Brown failure criterion is presented as Eq. (1).

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^a \tag{1}$$

The expressions of m_b, s and a are

$$m_b = m_i\exp\left(\frac{GSI - 100}{28 - 14D}\right) \tag{2}$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \tag{3}$$

$$a = \frac{1}{2} + \frac{1}{6}(e^{-GSI/15} - e^{-20/3}) \tag{4}$$

where σ_1 and σ_3 are respectively the major and minor principal stresses at failure, σ_c is the uniaxial compressive stress of the rock, GSI is the geological strength index which represents the integrity of the rock mass, D is the disturbance coefficient of the rock mass which varies from 0 for undisturbed rock mass to 1.0 for heavily disturbed rock mass, and m_i is the rock material constant.

In view of the fact that some classical theories are more convenient to be applied from the perspective of the normal and shear stresses, an exact mathematical relationship between Hoek-Brown failure criterion[15,16] and Mohr envelop is derived as

$$\tau = A\sigma_c\left(\frac{\sigma_n - \sigma_t}{\sigma_c}\right)^B \tag{5}$$

where σ_n and τ respectively represent the normal and shear stresses, A and B are dimensionless constants depending on the rock properties which have been specified in,[13] σ_t represents the tensile strength of the rock which can be obtained by

$$\sigma_t = \frac{\sigma_c}{2}\left(m_b - \sqrt{m_b^2 + 4s}\right) \tag{6}$$

As interpreted by Hoek and Brown, the equation is not so practical, since an additional statistical curve fitting process must be conducted in contrast with Eq. (1).

3. Discussions of the original approach of Fraldi and Guarracino[7]

The basic idea of the original approach proposed by Fraldi and Guarracino[7] can be sketched out as follows. According to the upper bound theorem of limit analysis, the potential collapsing block is investigated by means of a kinematical approach.[17–19] Subsequently a functional differential equation is derived and solved with the help of variational principle. to determine t The analytical solution of $f(x)$ is determined which outlines the shape of the potential collapsing block as shown in Fig. 1(a). L represents the half width of collapsing block, H represents the height of it and h represents the burial depth of the rectangular tunnel.

According to the ideal plastic materials, the plastic potential function Ω of Hoek-Brown rock mass can be expressed as

$$\Omega = \tau - A\sigma_c\left(\frac{\sigma_n - \sigma_t}{\sigma_c}\right)^B \tag{7}$$

Judging from the associated flow rule, the plastic strain increment is proportional to the stress gradient of plastic potential function.

$$\begin{cases} \dot{\varepsilon} = \lambda\frac{\partial\Omega}{\partial\sigma_n} = -\lambda AB\left(\frac{\sigma_n - \sigma_t}{\sigma_c}\right)^{B-1} \\ \dot{\gamma} = \lambda\frac{\partial\Omega}{\partial\tau} = \lambda \end{cases} \tag{8}$$

where λ is the plastic multiplier.

As suggested in,[7] the plastic strain rate components can be written in another form from the perspective of kinematically admissible velocity field.

$$\begin{cases} \dot{\varepsilon} = \frac{v_0}{t}\cos\alpha = \frac{v_0}{t\sqrt{1 + f'(x)^2}} \\ \dot{\gamma} = -\frac{v_0}{t}\sin\alpha = -\frac{v_0 f'(x)}{t\sqrt{1 + f'(x)^2}} \end{cases} \tag{9}$$

where v_0 is the velocity of collapsing block, t is the thickness of the plastic yield surface, α is the angle between the positive x-axis direction and the tangential direction.

In fact, it is implied in Eq. (9) that the velocity of the particle on yield surface is in accordance with that of collapsing block. However, such a practice seems not so reasonable considering the fact that the velocity of the particle on yield surface v should deviate from the velocity discontinuity with an angle φ, namely the internal friction angle, as shown in Fig. 1(b).[20,21] Provided that η is the angle between v and the vertical direction and take the positive value along the positive x-axis, a more convincing presentation can be gained in Eq. (10).

$$\begin{cases} \dot{\varepsilon} = \frac{v}{t}\cdot\cos(\alpha + \eta) = \frac{v}{t}\cdot(\cos\alpha\cos\eta - \sin\alpha\sin\eta) \\ \dot{\gamma} = -\frac{v}{t}\cdot\sin(\alpha + \eta) = -\frac{v}{t}\cdot(\sin\alpha\cos\eta + \cos\alpha\sin\eta) \end{cases} \tag{10}$$

Likewise the relationship between v_0 and v can be derived from Fig. 1(b).

$$v_0 = v\cos\eta \tag{11}$$

An equivalent form of Eq. (10) is obtained as follows

$$\begin{cases} \dot{\varepsilon} = \frac{v_0}{t}\cdot[1 - f'(x)\tan\eta]\cdot\frac{1}{\sqrt{1 + f'(x)^2}} \\ \dot{\gamma} = -\frac{v_0}{t}\cdot\left[1 + \frac{\tan\eta}{f'(x)}\right]\cdot\frac{f'(x)}{\sqrt{1 + f'(x)^2}} \end{cases} \tag{12}$$

Obviously, it can be speculated that the original approach by Fraldi and Guarracino[7] is just a special case when $\eta = 0$ which actually increases the internal friction angle. To get more credible results and extend the scope of its application, cases when $\eta \neq 0$ should be explored thoroughly. In fact, it is really a tough task to handle this problem as the variational procedure becomes extremely complex by substituting Eq. (12) into the original approach. Notice that η is also a function of x due to the variation of α and φ. As a result, a simplification technology is more desirable to be employed to modify the original approach aiming at a better approximation of the actual results.

4. A modified approach based on Fraldi and Guarracino[7]

To balance the credibility of the results and the feasibility of the variational procedure, a simplification technology is introduced by straightforwardly determining the value of $f'(x)$ of the middle term in Eq. (12). As is shown in Fig. 2, $f'(x)$ is simplified as $\tan\alpha_0$ (the tangent value of α_0) which can be solved by two arbitrary points P_1, P_2 on the curve of $f(x)$. Likewise η is considered as a constant as well. Suppose