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Modeling ground displacement above reservoirs undergoing fluid withdrawal/injection based on an ellipsoidal inhomogeneity model

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ABSTRACT

We propose a procedure to efficiently monitor the geomechanical behavior of underground porous reservoirs undergoing fluid withdrawal/injection. We apply the inhomogeneity problem and invert geodetic data on subsidence/uplift to calibrate and specify parameters used for geomechanical models. Based on pressure changes, we first calculate the poroelastic strain of the reservoirs, then we test it assuming uniaxial deformation on a three-dimensional ellipsoidal inhomogeneity with different semi-axes, in a full space isotropic elastic domain and finally we apply a simple half space approximation to account for the tractions-free ground surface. Despite its simplicity and the initial full space approximation, the semi-analytical model derived in this study replicates well poroelastic strains inferred by geodetic data or obtained with other analytical and numerical techniques. Our model is a good approximation for predicting the profile of surface deformation (i.e. geodetic strain) and, once calibrated, its magnitude.

1. Introduction

Reservoir compaction, land subsidence, triggering of landslides and earthquakes have been attributed to poroelastic stress changes generated by fluid injection or extraction in/from different hydrocarbon fields.[1–4]

Fluid extraction or injection from/into a reservoir causes pore pressure and effective stress changes due to poroelastic coupling, both in the reservoir and the sealing/encasing rocks. The variations in pore pressure and effective stress cause compaction or dilation in the reservoir generating subsidence or uplift of the ground surface and, in more extreme cases, damages to infrastructures and induced seismicity. In the last decade, public opinion and administrative authorities have become sensitive to these latter risks, which are directly related to an increasing demand for energy (hydrocarbon production) and the strategic need to store gas in the subsurface at different locations for controlling demand, price, and international crisis.[5,6]

Release of permissions for hydrocarbon production and storage as well as reservoir management, today rely even more than in the past on predictive tools based on geomechanical models. Different types of geomechanical models, based on analytical, semi-analytical/closed form, and numerical techniques, have been developed to estimate deformation in the reservoir and at the ground surface induced by fluid pressure change and for resolving inverse problems (for parameters estimation).

Assuming mechanical homogeneity between reservoir and bounding rocks, Geertsma[1] developed an uniaxial model for reservoir compaction and related land subsidence based on the *nuclei of strain* concept[7] in an elastic half-space. Segall,[3] by using the Eshelby's inclusion theory, used the concept of “transformational strain” for calculating external stresses and deformation derived by a uniform fluid withdrawal from a reservoir in an half-space under plane strain conditions[3] and with an axisymmetric shape[8,9] assuming reservoirs having the same mechanical properties of the surrounding rocks. Following the work of Segall[8], Du and Olson[10] constructed a numerical model to predict surface subsidence and reservoir compaction, and Vasco et al.[11] analyzed surface tilt generated by fluid injection and soil consolidation to estimate the surface deformation.

Other methods have been used to account for different reservoir geometries and for the effects of variations in mechanical parameters between reservoir and surrounding rock. For example, Rudnicki[12] solved for the problem of a poroelastic reservoir, mechanically different from the surrounding rocks, using the theory developed by Eshelby[13] and considering the reservoir as an elliptical inhomogeneity in an elastic full space. Soltanzadeh et al.[14] solved the problem of a lateral infinite poroelastic inhomogeneity with elliptical cross-section under plane strain conditions in a full space.

The main limitation of the previous analytical methods, is the assumption of simplified geometry, i.e. ellipsoidal reservoir with

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only one semi-axis different than the others or infinite lateral extent of the reservoir, or the consideration of reservoir properties being identical to those of the surrounding material; our method overcomes some of these geometric limitations.

One limitation in our method is the full-space approximation, which neglects the effect of the free boundary surface; the induced strain from a constant eigenstrain in the inclusion of a semi-infinite medium, in fact, is not uniform. According to Rudnicki,[12] however, the full-space plane strain solutions can be good approximations for relatively deep reservoirs, for reservoirs with thickness much less than their lateral extent, and where the assumption of uniaxial strain in the reservoir is valid, as long as the shear modulus of the reservoir is not much larger than that of the surrounding material.

Davies[15] has shown that the elastic field due to non-uniform temperature or a coherently misfitting inclusion in a semi-infinite region can be simply derived from the corresponding field in an infinite region; in particular he showed that displacement of a free surface is the same as that of the equivalent plane in an infinite solid multiplied by a factor $4(1-\nu)$.

The objective of our work is to develop a new simple method for first assessment of reservoir deformation. Our main contribution is to calculate the eigenstrain from pressure changes by taking advantage of a semi-analytical model developed by Meng et al. in Ref. 16 which allows to calculate strains, stresses, and displacements for an ellipsoidal inhomogeneity in an infinite isotropic full space, with complex geometries for the ellipsoid (i.e. three different semi-axis), through the equivalent inclusion method based on Refs. 13 and 17.

After an overview of the poroelastic inhomogeneity problem, we describe a method to monitor the strains associated to underground porous reservoirs undergoing fluid withdrawal/injection. Then, we test this method by comparing our modeled land subsidence with published examples of two reservoirs, the Lacq field[9] and the Lombardia field,[18] where surface deformation has been derived from field surveys as well as from analytical and numerical modeling approaches. We show that the inhomogeneity problem can rapidly, accurately, and efficiently model the deformation profile of the ground surface during production or injection of fluids in “real world” poroelastic reservoir.

2. Methods

The method that we follow in our analysis is based on the solution of the inhomogeneity problem under an uniaxial strain approximation. Our major contribution is to apply this method to real-world reservoir fluids injection/production problems. All calculations have been performed using the Matlab™ software. The code used is a modification of Ref. 16.

2.1. The inhomogeneity problem

Consider an inclusion Ω embedded in an infinite homogeneous isotropic elastic medium D (Fig. 1). The inclusion undergoes an eigenstrain ε^*. The induced elastic fields caused by eigenstrain ε^*_{ij} in the inclusion can be written as[17]

$$u_i(\mathbf{x}) = -\int_\Omega C_{jlmn}\varepsilon^*_{mn}(\mathbf{x}')G_{ij,l}(\mathbf{x}-\mathbf{x}')\mathrm{d}\mathbf{x}', \tag{1}$$

$$\varepsilon_{ij}(\mathbf{x}) = -\frac{1}{2}\int_\Omega C_{klmn}\varepsilon^*_{mn}(\mathbf{x}')\left\{G_{ik,lj}(\mathbf{x}-\mathbf{x}') + G_{jk,li}(\mathbf{x}-\mathbf{x}')\right\}\mathrm{d}\mathbf{x}', \tag{2}$$

$$\sigma_{ij}(\mathbf{x}) = -C_{ijkl}\left\{\int_\Omega C_{pqmn}\varepsilon^*_{mn}(\mathbf{x}')\,G_{kp,ql}(\mathbf{x}-\mathbf{x}')\mathrm{d}\mathbf{x}' + \varepsilon^*_{kl}(\mathbf{x})\right\} \tag{3}$$

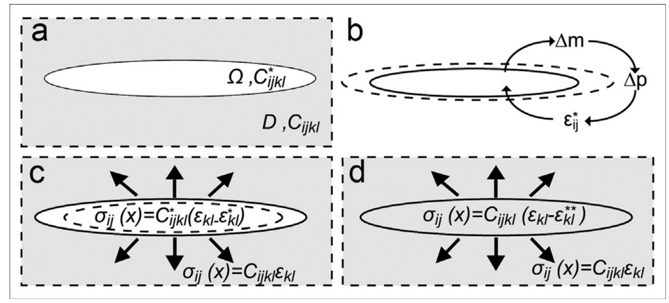

Fig. 1. The inhomogeneity problem and its equivalent inclusion solution. (a) Ellipsoid volume Ω, with a different elastic moduli C^*_{ijkl} (inhomogeneity) from the encasing full space elastic domain (the matrix) D (C_{ijkl}). (b) The inhomogeneity contracts/dilates by ε^*_{ij} without the constraint of the matrix. (c) Stresses are applied to the inhomogeneity in order weld it to the matrix (d). The resulting stresses are solved by the equivalent inclusion method.

where $\varepsilon^*_{ij}(\mathbf{x})$=0 for $\varepsilon^*_{mn}(\mathbf{x}')\in D$ and C_{ijkl} are the elastic constants of the material, for both inclusion and matrix. The Green's function $G_{ij}(\mathbf{x}-\mathbf{x}')$ gives the displacement component in the x_i direction at a generic point $\mathbf{x}$ when a un$_i$t body force in the x_i direction is applied to a point $\mathbf{x}'$ in the infinite medium.

Unless specifically indicated, the conventional summation convention for the repeated indices is used, whereby repeated indices indicate summation over the values 1, 2, 3, and indices preceded by a comma denote differentiation with respect to the Cartesian coordinates corresponding to the index following the comma.

It has been shown by Eshelby[13] that for an ellipsoidal inclusion with uniform eigenstrain, the induced strain in the inclusion is uniform and can be expressed as follows:

$$\varepsilon_{ij}(\mathbf{x}) = S_{ijkl}\,\varepsilon^*_{kl} \tag{4}$$

where S_{ijkl} is called the Eshelby's tensor, which is a function of the inclusion shape and of the elastic properties of the matrix (in the case of an isotropic matrix, it is a function of the Poisson’s ratio only), with $S_{ijkl} = S_{jikl} = S_{ijlk}$. Detailed expressions of Eshelby's tensor for various ellipsoidal inclusions have been reported in Ref. 17.

If the sub-domain Ω has elastic properties differing from those of the rest of the domain $D-\Omega$ (matrix) than it is called an inhomogeneity. The inhomogeneity is called an inhomogeneous inclusion when it undergoes its own eigenstrain. Eshelby[13] pointed out that the problem of an ellipsoidal inhomogeneous inclusion can be transformed into an equivalent inclusion problem when the correct equivalent eigenstrain is chosen. Consider an ellipsoidal domain Ω with elastic moduli C^*_{ijkl} embedded in an infinite homogeneous medium with the elastic moduli C_{ijkl} (Fig. 1a) undergoes an eigenstrain ε^* (Fig. 1b). Because the eigenstrain is considered stress free (i.e. without the constraint of the matrix), we need to subtract it from the total strain when we calculate the stress in the inhomogeneity or inclusion (Fig. 1c).

The induced stress can be found as follows:

$$\sigma_{ij}(x) = C^*_{ijkl}\left(\varepsilon_{kl} - \varepsilon^*_{kl}\right)\ in\ \Omega,\ \ \sigma_{ij}(\mathrm{x}) = C_{ijkl}\varepsilon_{kl}\ in\ D-\Omega \tag{5}$$

In the equivalent inclusion method, the inhomogeneity is replaced by an inclusion in the homogeneous medium with an equivalent eigenstrain ε^{**}_{ij} (introduced here as fictitious one just for the simulation). Then the induced stress can be obtained as follows:

$$\sigma_{ij}(x) = C_{ijkl}(\varepsilon_{kl} - \varepsilon^{**}_{kl})\ in\ \Omega\ \sigma_{ij}(X) = C_{ijkl}\varepsilon_{kl}\ in\ D-\Omega \tag{6}$$

This fictitious eigenstrain ε^{**}_{ij} (for the equivalent inclusion) can be obtained by solving the equivalence between the stresses in the inhomogeneity (5) and the stresses in the equivalent inclusion (6)