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Improved analytical solutions for the response of underground excavations in rock masses satisfying the generalized Hoek–Brown failure criterion

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ABSTRACT

Analytical solutions for tunnel design are widely used in practical engineering, as they allow a quick analysis of design issues such as estimation of support requirement. In recent years, several papers analyzing the behavior of rock masses that obey the conventional or generalized Hoek–Brown criterion have been published. This article presents a complementary analysis that includes a new normalization of the generalized Hoek–Brown failure criterion, complete solutions for associated and non-associated flow rules, with some new closed-form solutions in the latter case, and in-depth considerations regarding intermediate stresses and edge effects. The results obtained show full agreement with existing solutions in the literature, when possible, and with numerical finite element models in cases that had not been treated previously.

1. Introduction

Major civil engineering projects such as tunnels at depth pose significant challenges to soil and rock mechanics engineers alike. During the design stage, these types of projects require systematic treatment by engineers, who, among others, have to develop a geotechnical model of the site, optimize the design, and assess the uncertainties and risks in the short- and long-term life of the project[1]. To do this, the geotechnical engineer seeks to identify the physical properties that significantly influence the mechanical behavior of the rock mass, based on site surveys, core analyses, and laboratory tests on rock samples. Then, the engineer relies on both empirical and theoretical approaches to include all of this technical data into a calculation process for design. The ultimate goal is to determine a safe and financially acceptable solution that is compatible with all the constraints applicable to the project. In the end, the construction phase provides an insight into the real behavior of the ground and may allow the geotechnical engineer to identify defects in the model, adjust the design, and gain experience.

The case of tunnels fits this description well since, when they are not self-stable, underground works are characterized by a significant interaction between the stabilizing structures and the surrounding geological material. Consequently, the relevance of the geotechnical model depends strongly on its ability to take into account two major uncertainties: variability in material properties and representativeness of calculation processes. In this context, simplified design methods such as convergence-confinement are worth using, as they allow a quick and reasonable assessment of tunnel support and may be used for sensitivity studies. As such, they can be part of improvements in tunnel design, as they facilitate exploration of the possible range of stability conditions around the excavation.

Many authors have proposed analytical or semi-analytical solutions for establishing the ground characteristic curves that describe the behavior of a rock mass affected by tunnel excavation. Most often, these solutions rely on hypotheses that allow an axisymetric approach (circular tunnel at great depth in an isotropic and homogeneous material, with an isotropic initial stress field and conditions for plane strain), and tunnel excavation is modeled by decreasing a fictitious internal pressure applied to tunnel periphery. These solutions may consider various material models (linear or non-linear elasticity, failure criteria, etc.). For instance, more than 30 years ago, Brown et al.[2] could already refer to 22 significant contributions on this subject made since Fenner's early

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work[3]. In recent years, significant work has been undertaken to obtain more ready-to-use mathematical formulations for ground characteristic curves, including the well-known generalized Hoek–Brown failure criterion. These developments are of particular interest because the Hoek–Brown criterion may be used in a wide range of situations for underground works, including fractured hard rocks but also materials such as hard soils – soft rocks (see Refs. 4 and 5 for instance).

Generally speaking, the plastic formulations used in these theoretical developments can be divided into those for which stress–strain relationships are based on the incremental or flow theory of plasticity (IS, for "incremental strain") and those based on the deformation or total strain theory of plasticity (TS, for "total strain"). The IS formulations relate stress to plastic strain increments and therefore involve 'time variables' (through kinematic parameters) in addition to usual space variables. As a result, the IS formulation requires solving partial differential equations or alternatively, in problems such as the one to be solved in this article, transformation of these partial differential equations into total differential equations that can be solved in closed-form. In contrast, the TS formulations relate stresses to total plastic strains and do not involve a 'time variable': as a result, for problems involving one space variable only, as in the case of the problem addressed in this paper, they require solving total differential equations only (from a mathematical point of view, the TS formulation is usually significantly simpler than the IS formulation). Although both types of formulations yield the same results in simple problems involving mononotic progression of loading/unloading, as in the case of the problem addressed in this work, the IS formulation is supposed to be more rigorous and more general than the TS formulation because it can be applied to solve plastic problems which do not necessarily involve mononotic progression of loading/unloading. Moreover, a TS formulation may be difficult to express in the case of non-linear flow rules (such as associated Hoek–Brown).

Subsequent to the first analysis from Brown et al.[2], in which several simplifications were considered (no elastic strain within the plastic radius for instance), three important results can be cited. First, Carranza-Torres et al.[6] and Carranza-Torres[7] described a full and rigorous solution with both original and generalized Hoek–Brown criteria, with associated or non-associated flow rules, and perfectly plastic or brittle-plastic behaviors. They used the IS formulation for plasticity, but obtained total differential equations with ingenious variable changes. Shortly thereafter, Sharan[8–10] proposed an alternative solving process with a TS formulation, limited to the case of a Mohr–Coulomb flow rule, and yielding comparable results. Finally, Serrano et al.[11] described an additional solving method using a general expression of the failure criterion (written in terms of Lambe variables) and of a non-associated flow rule (including a stress-dependant dilatancy). They used TS plasticity, and their results matched well with the aforementioned works in the case of a Hoek–Brown failure envelope.

However, despite the reliability of this scientific work, the authors noted that improvements could still be suggested for ground characteristics curves in a rock mass obeying the generalized Hoek–Brown criterion. In particular, a new normalization of the criterion leading to simplified equations is presented in the next sections. Moreover, the problem of edge effects on the failure surface is fully addressed, allowing one to evaluate its influence on the description of rock mass behavior. A new closed-form solution is also presented in the case of an associated flow rule, using IS plasticity and variable changes.

2. Description of the problem and governing equations

The calculation process described below relies mainly on the approach proposed in Refs. 6 and 7, with several improvements as mentioned in the previous section. The hypotheses are as follows.

2.1. Problem description in the context of the convergence-confinement method

The convergence-confinement method (also called the method of characteristic curves) is aimed at describing as precisely as possible the principle of interaction between a rock mass and a tunnel support. It allows for taking into account the "work" of the rock mass, during the design process, since this work contributes to the stability of the excavation. The description of ground behavior is based on a direct integration of mechanical equations, thus requiring some simplifying hypotheses.

First, the problem has to be studied in an axisymetric configuration, which implies several assumptions. The rock mass must be considered as homogeneous and isotropic, with isotropic initial stress conditions. The excavation must be circular and at great depth (at least 10 times the tunnel radius), so as to be able to disregard the influence of gravitational and side effects. Moreover, the tunnel has to be long enough to assume a plane strain state (i.e. $\varepsilon_x = 0$, with x being oriented towards the longitudinal axis of the tunnel). Using these hypotheses, the principle of the convergence-confinement method can be summarized according to the diagram shown in Fig. 1.

During the excavation process, convergence begins ahead of the tunnel face (see Ref. 13 or 14, for example) and maximum displacement is not obtained immediately after the excavation phase. This variation of convergence with advance is usually referred to as the longitudinal displacement profile LDP. For a non-supported tunnel, the convergence-confinement method represents the effect of the advancing tunnel face via a fictitious internal pressure σ_i applied to the tunnel walls which decreases progressively from the initial stress in situ σ_0 to zero when the plane strain state is obtained. The ground reaction curve (or convergence curve) GRC, which can be calculated using closed-form

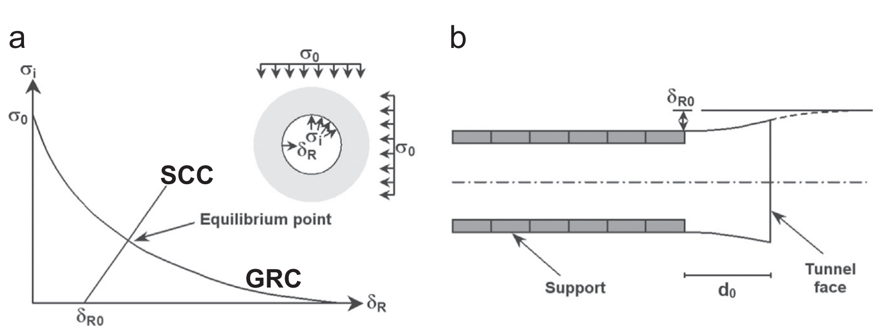

Fig. 1. Schematic representation of the basis of the convergence-confinement method (after Ref. 12): (a) shows how the equilibrium point is obtained; while (b) represents the convergence process due to tunnel face advance and the definition of parameters d_0 and d_{R0}.