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Tensile failure criterion for transversely isotropic rocks

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ABSTRACT

A common problem encountered in rock engineering involves the specification of strength anisotropy in transversely isotropic rocks such as slate and schist. The strength anisotropy under compression has been relatively well established, whereas the formulation of the tensile strength anisotropy has received less attention due to the difficulty of experimental validation. In this paper, a recently developed 3-D tensile failure function for transversely isotropic rocks is further examined. The failure condition incorporates three strength parameters and employs a 2nd order tensor, whose principal directions are those of material fabric, to describe the spatial variation of the tensile strength. It is shown that both the single plane of weakness theory of tensile failure and Nova and Zaninetti's failure condition are two special cases of the 3-D criterion in the context of the spatial distribution of the tensile strength. A procedure for the identification of the strength parameters is outlined. The direct tensile tests on the transversely isotropic samples with various orientations are simulated by invoking the critical plane approach with the 3-D tensile failure function. The loading conditions considered in the simulation include uniaxial tension, triaxial tension, triaxial extension and true triaxial extension. The ultimate value of the axial tensile stress at failure and the associated orientation of the failure plane are discussed.

1. Introduction

Many rock materials, such as sedimentary and metamorphic rocks, display distinct anisotropy in strength due to the inherent rock fabric or the alignment of stress-induced defects to a preferential orientation. Hence, considering the fact that rock engineering activities, such as tunneling, rock drilling and slope cutting, frequently take place in anisotropic rock formations, the accurate assessment of the strength characteristics related to anisotropic rock is of special importance. Among the different forms of anisotropy, the transverse isotropy is the simplest, but perhaps most important one, since the periodic planar structures are embedded as foliation or bedding in many metamorphic and sedimentary rocks, and they act as the weakness planes causing various stability problems.

In view of this, the topic of strength anisotropy in transversely isotropic rock has drawn much attention in rock mechanics since the early 1960s. Jaeger's theory of single plane of weakness (SPW)[1] was among the first which tried to describe the strength anisotropy in a theoretical manner. In the SPW theory, both the friction angle and the cohesion of the weakness plane are different from the remaining intact material, so that the spatial distribution of the strength parameters is not continuous. In the same reference, Jaeger also suggested a theoretical model with continuously variable cohesion. Since then, a lot of experimental research has been conducted in order to understand the failure behavior of the transversely isotropic rocks, such as slate, shale, schist and phyllite.[2–8] The experimental works have confirmed that the strength of the transversely isotropic rocks is significantly affected by the loading direction with respect to the bedding or foliation planes. In addition, the experimental data appearing in Donath[2] and Attewell and Sandford[4] indicate that the orientation of the failure plane is also closely related to the orientation of the weakness planes in relation to the loading direction.

Along with the experimental work, a number of failure criteria for transversely isotropic rocks have been proposed. A comprehensive review on this topic, which includes a study on predictive abilities of several representative criteria, has been provided by Duveau et al.[7] Most of the existing experimental and theoretical work on the assessment of the conditions at failure has been focused on the loading condition of conventional triaxial compression, which lessens their practical usefulness since the in-situ stress condition is, in general, true triaxial. In the context of the theoretical studies, more rigorous transversely isotropic failure criteria, formulated for the general 3-D stress conditions, have been suggested by some researchers.[9–11] However, the disadvantage of these 3-D formulations lies in the fact that they incorporate additional strength parameters, the identification

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procedures of which are in general quite complicated, requiring a very sophisticated experimental program.

As briefly reviewed above, to date the shear failure behavior of transversely isotropic rocks under compression has been fairly well understood from both the experimental and theoretical viewpoints. On the other hand, the formulation of the tensile strength anisotropy has been less developed in spite of its dominant role in predicting the initiation of tensile cracking in sedimentary and metamorphic rocks caused by various activities such as hydraulic fracturing, slope cutting, long span tunneling and rock blasting. Experimental evidence on the orientation dependency of tensile strength in the transversely isotropic rocks can be found in the literature.[12–14] Hobbs[15] showed that the variation of the tensile strength of laminated rock can be predicted by the Griffith crack theory.[16] Hobbs' tensile failure condition is, however, restricted to uniaxial loading and fails to provide the orientation of the failure plane. Later, Nova and Zaninetti[13] proposed an anisotropic tensile failure criterion for an orthotropic medium by invoking the notion of tensile strength tensor, which has similar properties to the stress tensor. Although Nova and Zaninetti's failure condition is more advanced in that the formulation is theoretically straightforward and considers the general stress states, its implementation in true triaxial conditions seems to be very complicated. More recently, another 3-D formulation was proposed by Lee et al.,[17] which is based on the critical plane framework.[18] In this approach, the Rankine's tension cut-off criterion has been extended for orthotropic medium by employing the concept of a 2nd order tensor which characterizes the directional bias in the spatial distribution of the tensile strength.

In this paper, we further investigate the tensile failure criterion proposed by Lee et al.[17] and prove that both the SPW theory formulated for the tensile stress regime and Nova and Zaninetti's tensile failure condition are the special cases of the Lee et al.'s criterion. Moreover, an identification procedure for the associated strength parameters, which uses the data obtained from the direct tensile tests on differently oriented samples, is proposed. At the same time, the efficiency of this approach is demonstrated by simulating the direct tensile tests reported by Nova and Zaninetti.[13] Finally, the tensile failure behavior of the transversely isotropic rocks in the general 3-D stress regime, including the loading conditions of triaxial tension, triaxial extension and true triaxial extension, is simulated and the related findings are discussed.

2. Theoretical treatments of tensile strength anisotropy

2.1. Theory of a single plane of weakness

By analogy to the Jaeger's original work on shear failure,[1] the tensile equivalent of the single plane of weakness (SPW) theory could be formulated by assuming that every physical plane with the exception of weakness plane has identical tensile strength. Since the tensile strength across the weakness plane is, in general, much lower than that of intact rock material, the tensile fracturing is likely to occur along this plane provided that its inclination is less than a critical value. In this respect, for a rock sample subjected to confining pressure σ_3 at different orientations of the weakness plane d as shown in Fig.1, the tensile failure will occur when the normal stress across the weakness plane reaches a critical value T_w, which can be stated as

$$\sigma_{t(d)}\cos^2(d) - \sigma_3 \sin^2(d) = -T_w \tag{1}$$

so that

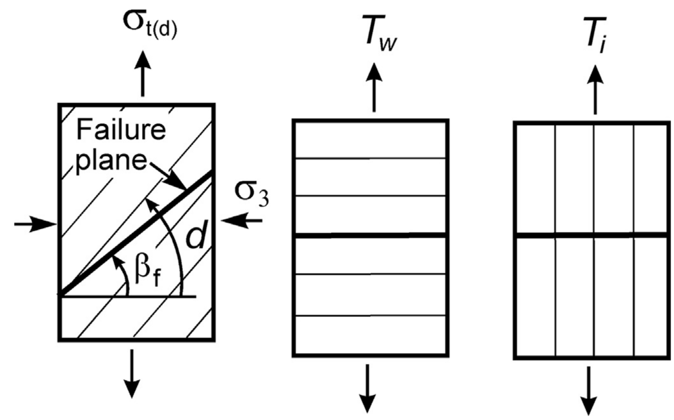

Fig. 1. Tensile strength of oriented samples.

$$\sigma_{t(d)} = \sigma_3 \tan^2(d) + \frac{T_w}{\cos^2(d)} \tag{2}$$

This failure mode holds true for d less than a critical value d^*. For $d^* \le d \le 90°$, the tensile strength is equal to the tensile strength of intact rock material T_i and the failure plane is perpendicular to the loading direction, viz. $\beta_f = 0$.

By equating the right-hand side of Eq.(2) with T_i, the expression for the critical angle d^* is obtained as

$$d^* = \cos^{-1}\left(\sqrt{\frac{T_i + \sigma_3}{T_w + \sigma_3}}\right) \tag{3}$$

As an example, Fig. 2 shows the variation of axial strength $\sigma_{t(d)}$ and associated orientation β_f of the failure plane as a function of the inclination of the weakness plane at different values of T_i/T_w for $\sigma_3 = 0$. In this figure, $T_i/T_w = 1$ denotes an isotropic sample. It is evident that the anisotropy in axial tensile strength becomes more pronounced with increase of T_i/T_w. The tensile failure plane is parallel to the weakness plane if $d \le d^*$, whereas it makes a right angle with loading direction for d within the range $d^* < d \le 90^*$.

2.2. Failure criterion based on the tensile strength tensor

2.2.1. Evolution of tensile strength

Nova and Zaninetti[13] proposed a generalized tensile failure criterion by invoking the concept of a symmetric tensile strength tensor of order two to describe the orientation dependency of strength. In their formulation, the spatial distribution of tensile strength is defined by the projection of the tensile strength tensor $\mathbf{T}$ on the direction $\mathbf{n}$

$$T(\mathbf{n}) = (\mathbf{Tn})\cdot\mathbf{n} \tag{4}$$

where the scalar quantity $T(\mathbf{n})$ represents the tensile strength across a plane having the unit normal vector $\mathbf{n}$. For an orthotropic material, $\mathbf{T}$ has three independent eigenvalues which represent the tensile strengths in the principal material directions. In this case, the tensile strength tensor takes the form

$$\mathbf{T} = \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_2 & 0 \\ 0 & 0 & T_3 \end{bmatrix} \tag{5}$$

On the other hand, the number of distinct eigenvalues is reduced to two for transversely isotropic and one for isotropic material.

In order to illustrate the nature of the tensile strength distribution (4), consider a transversely isotropic sample with horizontal weakness planes, as shown in Fig. 3. Noting that the unit