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A microcracks-induced damage model for initially anisotropic rocks accounting for microcracks closure

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ABSTRACT

We formulate a new micromechanical damage model for anisotropic rocks. This model accounts not only for the coupling between material initial anisotropy and the damage-induced one, but also for the opening/closure status (the so-called unilateral effects) of evolving microcracks. A closed-form expression of the overall free energy of the microcracked medium is implemented in an appropriate thermodynamics framework to derive a complete damage model for initially anisotropic rocks. The salient features of this model are fully illustrated. Then, its capabilities are demonstrated through an application to a Taiwan argillite subjected to direct tensile loading (including off-axis ones) for which the damage model well captures experimental data (mechanical response, growing damage rocks strength). Finally, the response of the studied rock along a tensile loading followed by an unloading and a reloading in compression is provided in order to illustrate the so-called unilateral damage effects due to microcracks closure.

1. Introduction

The complex inelastic behavior of brittle rock-like materials under mechanical loading generally results from damage phenomena due to evolving microcracks. For the non-linear mechanical response of microcracked materials, Continuum Damage Mechanics (CDM, see, for instance, the textbooks of [1] and [2]) offers an appropriate theoretical framework. Since several decades, both phenomenological and micromechanical approaches of damage have been proposed. For continuum micromechanics, mention has to be made of several contributions dealing with effects of microcracking on materials properties (see, for instance, [3–6]). Formulation of isotropic or anisotropic damage by microcracks growth in rocks or concrete materials has been provided in several studies (see, for instance, [7–19]).

Despite their interest, the above cited models concern only materials which are initially isotropic (in their undamaged state). The purely macroscopic formulation of constitutive models coupling explicitly initial anisotropy and damage-induced one has been only investigated in few recent studies mainly devoted to brittle matrix composites; we refer here to Halm et al. [20] (see also [18] and [21]). Mention has also to be made of the purely macroscopic model for an initially anisotropic rock by Chen et al. [22]. Concerning micro–macro models, Baste et al. [23] and recently Monchiet et al. [24] proposed appropriate damage models which couple structural anisotropy and damage by microcracking. Yet, this class of models are limited to damage processes generated by open microcracks growth and need to be completed in order to properly account for microcracks closure.[1] To this end, Goidescu et al. [25] recently established closed-form expressions of the overall free energy of orthotropic materials weakened by microcracks, either open or closed. The present study takes advantage of these very recent results in order to formulate a complete model which fully couples initially anisotropy and evolving unilateral damage due to 2D systems of open or closed microcracks under frictionless conditions.

The paper is organized as follows. We briefly recall the closed-form expression of the macroscopic free energy which will play in the present study the role of a thermodynamics potential for the damaged material. The state laws derived from this potential provide the macroscopic stress as well as the damage energy release rate (thermodynamic forces conjugated to damage) as a function of the macroscopic strain and the damage variables. By adopting a discrete damage representation defined by the microcracks density parameter, we then propose a damage yield function based on the damage energy release

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[1] The corresponding unilateral effects are of paramount importance and necessary for quasi brittle geomaterials usually subjected to tensile loadings as well as compressive ones.

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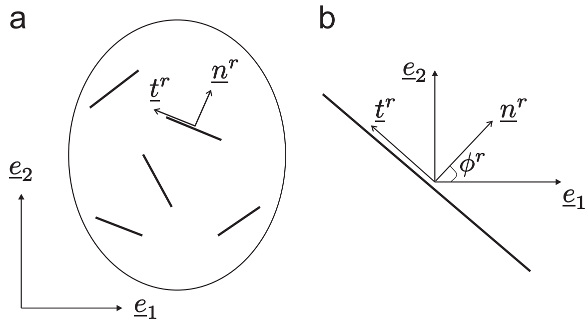

Fig. 1. (a) Representative volume element in 2D case; (b) crack coordinates system.

rate associated to each microcracks family. The corresponding damage surface is illustrated for various configurations of the microdefects system. Finally, we provide the damage evolution law by assuming normality rule and following classical thermodynamics-based procedure. This allows us to establish the complete rate formulation of the fully anisotropic constitutive damage law with account of microcracks closure. After a simple calibration step, the proposed model is assessed by comparing its prediction under tensile loading to available data on an argillite studied by Liao et al. [26]. Finally, the ability of the model to also account for microcracks closure effects is fully demonstrated in several cases.

Notations: Standard tensorial notations will be used throughout the paper. Lower underlined case letters will describe vectors, while bold script capital letters will be associated to second-order tensors and mathematical double-struck capital letters to fourth-order tensors. The following vector and tensor products are exemplified: $(\mathbf{A}\cdot\underline{b})_i = A_{ij}b_j$, $(\mathbf{A}\cdot\mathbf{B})_{ij} = A_{ik}B_{kj}$, $(\mathbb{A}:\mathbf{B})_{ij} = A_{ijkl}B_{kl}$, $(\mathbb{A}:\mathbb{B})_{ijkl} = A_{ijpq}B_{pqkl}$, $(\mathbf{A}\otimes\mathbf{B})_{ijkl} = A_{ij}B_{kl}$ and $(\mathbf{A}\overline{\underline{\otimes}}\mathbf{B})_{ijkl} = \frac{1}{2}(A_{ik}B_{jl} + A_{il}B_{jk})$. Einstein summation convention applied for the repeated indices and Cartesian coordinates are used. As usual, in the context of continuum micromechanics, small (respectively large) characters refer to microscopic (resp. macroscopic) quantities. $\mathbf{I}$ and $\mathbb{I}$ are, respectively, the second and fourth order identity tensors, the components of the former are represented by the Kronecker symbol (δ_{ij}) while for the latter one has $I_{ijkl} = (1/2)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$.

2. Overall free energy of a 2D anisotropic medium weakened by an arbitrarily oriented system of microcracks

2.1. Representative volume element (r.v.e.)

Micromechanical formulation of a brittle damage model requires first the determination of the effective properties of the microcracked material by using an homogenization procedure.

Let us consider a representative volume element *r.v.e.* Ω of the material (see Fig. 1(a)); this is constituted of an elastic orthotropic solid matrix s (with stiffness tensor $\mathbb{C}^s$ and occupying a domain Ω^s) and an arbitrarily oriented system of flat microcracks families (denoted r and occupying a domain Ω^r). The latter are assumed open or frictionless closed, non-interacting and in dilute concentration. This assumption allows us to fully develop a proper representation of the anisotropic multilinear response of weakened materials and provides basic solutions for future developments related to more complex configurations (including for instance interactions between microcracks[2]). Microcracks of the r^{th} family are characterized by their normal $\underline{n}^r$ and tangent $\underline{t}^r$ unit vectors, mean length $2l^r$ (the corresponding crack density is defined as $d^r = \mathcal{N}^r(l^r)^2$ in which $\mathcal{N}^r$ is the number of microcracks of this family per unit surface, see [3]).

This *r.v.e.* can be subjected either to uniform strain or uniform stress boundary conditions; the latter can take the form:

$$\boldsymbol{\sigma}(\underline{z})\cdot\underline{\nu}(\underline{z}) = \boldsymbol{\Sigma}\cdot\underline{\nu}(\underline{z}), \quad \forall \underline{z}\in\partial\Omega \tag{1}$$

in which $\boldsymbol{\sigma}$ denotes the microscopic stress field, $\boldsymbol{\Sigma}$ the macroscopic stress, $\underline{\nu}$ the outward unit normal to $\partial\Omega$ and $\underline{z}$ the vector position.

Let us recall that the present study deals with orthotropic materials weakened by arbitrarily oriented microcracks. The symmetry axes of the matrix correspond to an orthonormal basis $(\underline{e}_1, \underline{e}_2)$ (see Fig. 1) and its stiffness is given by

$$\mathbb{C}^s = a_1\mathbf{I}\otimes\mathbf{I} + a_2\mathbf{I}\overline{\underline{\otimes}}\mathbf{I} + a_3\mathbf{A}\otimes\mathbf{A} + a_4(\mathbf{A}\otimes\mathbf{I} + \mathbf{I}\otimes\mathbf{A}) \tag{2}$$

in which $\mathbf{A} = \underline{e}_1\otimes\underline{e}_1$ denotes the structural fabric tensor and where

$$\begin{aligned} a_1 &= \mathbb{C}^s_{2222} - 2\mathbb{C}^s_{1212}, \quad a_2 = 2\mathbb{C}^s_{1212} \\ a_3 &= \mathbb{C}^s_{1111} + \mathbb{C}^s_{2222} - 2\mathbb{C}^s_{1122} - 4\mathbb{C}^s_{1212}, \\ a_4 &= \mathbb{C}^s_{1122} - \mathbb{C}^s_{2222} + 2\mathbb{C}^s_{1212} \end{aligned} \tag{3}$$

and

$$\begin{aligned} &\mathbb{C}^s_{1111} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad \mathbb{C}^s_{2222} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \\ &\mathbb{C}^s_{1212} = G_{12}, \quad \mathbb{C}^s_{1122} = \mathbb{C}^s_{2211} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} \end{aligned} \tag{4}$$

E_1 and E_2 are the Young moduli in the symmetry axes of the material (respectively to $\underline{e}_1$ and $\underline{e}_2$), G_{12} is the shear modulus and ν_{12} and ν_{21} are Poisson ratios related to $(\underline{e}_1, \underline{e}_2)$ (Poisson ratios verify the relation: $E_1/\nu_{12} = E_2/\nu_{21}$). Equivalently, the compliance $\mathbb{S}^s = (\mathbb{C}^s)^{-1}$ of the matrix is defined as follows in the principal basis $(\underline{e}_1, \underline{e}_2)$ according to the Voigt notation:

$$\mathbb{S}^s = \begin{pmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{pmatrix}_{(\underline{e}_1,\underline{e}_2)}. \tag{5}$$

2.2. Thermodynamics potential of the anisotropic medium weakened by an arbitrarily oriented distribution of microcracks

The main homogenization procedure has been carried out by Goidescu et al. [25] who performed a direct microfractures mechanics-based analysis of the anisotropic damaged materials, in the spirit of the studies done in the context of isotropic matrix by [4,6,7,15–17] and others. The macroscopic thermodynamic potential of the anisotropic medium weakened by an arbitrarily oriented distribution of microcracks is then obtained as a function of the macroscopic strain tensor $\mathbf{E}$ (the observable state variable) and of the set of damage variables d^r (the internal state variables of the problem), noted $\underline{d}$, and associated to all microcracks family r ranging from 1 to $N = N_o + N_c$. Assuming a dilute concentration of microcracks, the solution of the homogenization problem comes to sum up the contributions of each family of parallel microcracks, namely

$$\begin{aligned} \Psi(\mathbf{E},\underline{d}) = {} & \frac{1}{2}\mathbf{E}:\mathbb{C}^s:\mathbf{E} \\ & - \sum_{r=1}^{N_o} d^r\left\{H^r_{nn}(\mathbf{N}^r:\mathbf{E})^2 + 2H^r_{nt}(\mathbf{N}^r:\mathbf{E})(\mathbf{T}^r:\mathbf{E}) + H^r_{tt}(\mathbf{T}^r:\mathbf{E})^2\right\} \\ & + \sum_{r=1}^{N_c} \frac{d^r}{H^r_{nn}H^r_{tt} - H^{r2}_{nt}}\left\{\begin{array}{l} H^r_{nn}H^{r2}_{nt}(\mathbf{N}^r:\mathbf{E})^2 + 2H^{r3}_{nt}(\mathbf{N}^r:\mathbf{E})(\mathbf{T}^r:\mathbf{E}) \\ + H^r_{tt}(2H^{r2}_{nt} - H^r_{nn}H^r_{tt})(\mathbf{T}^r:\mathbf{E})^2 \end{array}\right\} \end{aligned} \tag{6}$$

where N_o represents the number of open microcracks family and N_c the number of closed cracks. One has $\mathbf{N}^r = \mathbb{C}^s:(\underline{n}^r\otimes\underline{n}^r)$ and

[2] This could be done for instance by considering a Mori–Tanaka like homogenization scheme.