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An experimentally constrained constitutive model for geomaterials with simple friction–dilatancy relation in brittle to ductile domains

Daniel Mas, Alexandre I. Chemenda *

Université de Nice Sophia-Antipolis, CNRS, Observatoire de la Côte d'Azur, Géoazur, 250 av Einstein, 06560 Valbonne, France

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ABSTRACT

Complexity of the mechanical behavior of geomaterials makes it very difficult to formulate constitutive models (both at micro- and macro-scales) valid for different loading conditions and deformation regimes. To make progress in understanding this complexity, we take advantage of the large set of data for the synthetic rock analog GRAM1, a granular, frictional, dilatant, and cohesive material formed of bonded rigid particles. We use also data from literature for two real rocks. All data are from conventional triaxial tests conducted for a wide range of confining pressures covering the material behavior from brittle fracturing to ductile flow. The data processing allowed to define both the yield function and the inelastic volume strain as functions of the mean stress σ_m and the accumulated inelastic strain $\overline{\gamma}^p$. The internal friction coefficient and dilatancy factor calculated from these functions were shown to be different but evolving very similarly with σ_m and $\overline{\gamma}^p$ for all the three materials. This allowed to relate the yield and plastic potential functions and thereby to complete the constitutive formulation within the framework of the classical elastoplasticity theory. The obtained results are also used to elucidate the relation between the yield surface and failure envelope as well as the meaning of the internal friction coefficient derived from the failure envelope which is routinely used in geomechanical applications and which is very different from the internal friction coefficient derived from the yield function.

1. Introduction

The complexity of the mechanical behavior of geomaterials makes it very difficult to formulate mechanical (constitutive) models particularly those valid for a wide range of loading conditions. Most models are phenomenological, formulated in the framework of the elastoplasticity theory (e.g., [1–12]). The problem with these models is that they become progressively more complicated and include a considerable number of parameters (ten and more) when trying to approach real behavior of materials. The physical meaning of these parameters is generally unclear. This forces the research to take into account in one way or another the micromechanical processes in defining the mesoscale and macroscale constitutive laws of geomaterials based on various approaches, hypotheses, and assumptions. There exists extensive literature from different communities [13–22], to mention only a few papers, most of which are on the flow of cohesionless granular materials. The hope is that the complex real behavior of geomaterials can be predicted from simple microscale laws involving a small number of physically meaningful parameters that can be determined from a few relatively simple experimental tests. This hope however does not seem to have been fulfilled. To fit the experimental data, complex microscale constitutive laws have to be introduced that are also phenomenological in nature and are frequently taken in the same form as macro constitute laws of traditional elastoplasticity (e.g., [19,23,24]). Doing this inevitably increases the number of the model parameters. Starting from 6 parameters for the standard Discrete Element Model (DEM), D'Adetta and Ramm [23], for example, have gone up to 9, 13, and 18 independent parameters in more advanced models. One observes hence the same tendency towards complication as in classical elastoplasticity, but this time the uncertainty in constitutive relations is shifted from macro- to micro-scale. Thus, it must be recognized that none of the existing approaches is completely satisfactory. Progress in description of geomaterial properties requires both physics (particle)-based micromechanical efforts and phenomenological approaches. Both require as much as possible good quality experimental data characterizing geomaterials under different loading conditions and deformation regimes.

In this paper we process a large data set for three materials. The data analysis was done using the classical constitutive framework which includes three principal parts: the yield condition, expressed through the yield function $F(\sigma_{ij})$ that defines the limits of the elastic domain in the stress space; the flow rule defined by the plastic potential function $G(\sigma_{ij})$, and the hardening rules defining the evolution of $F(\sigma_{ij})$ and $G(\sigma_{ij})$ with progressive damage (inelastic deformation) of the material (σ_{ij} is the stress tensor, $i, j = 1, 2, 3$). To

* Corresponding author. Tel.: +33 4 8361 8661; fax: +33 4 8361 8610.
E-mail address: chem@geoazur.unice.fr (A.I. Chemenda).

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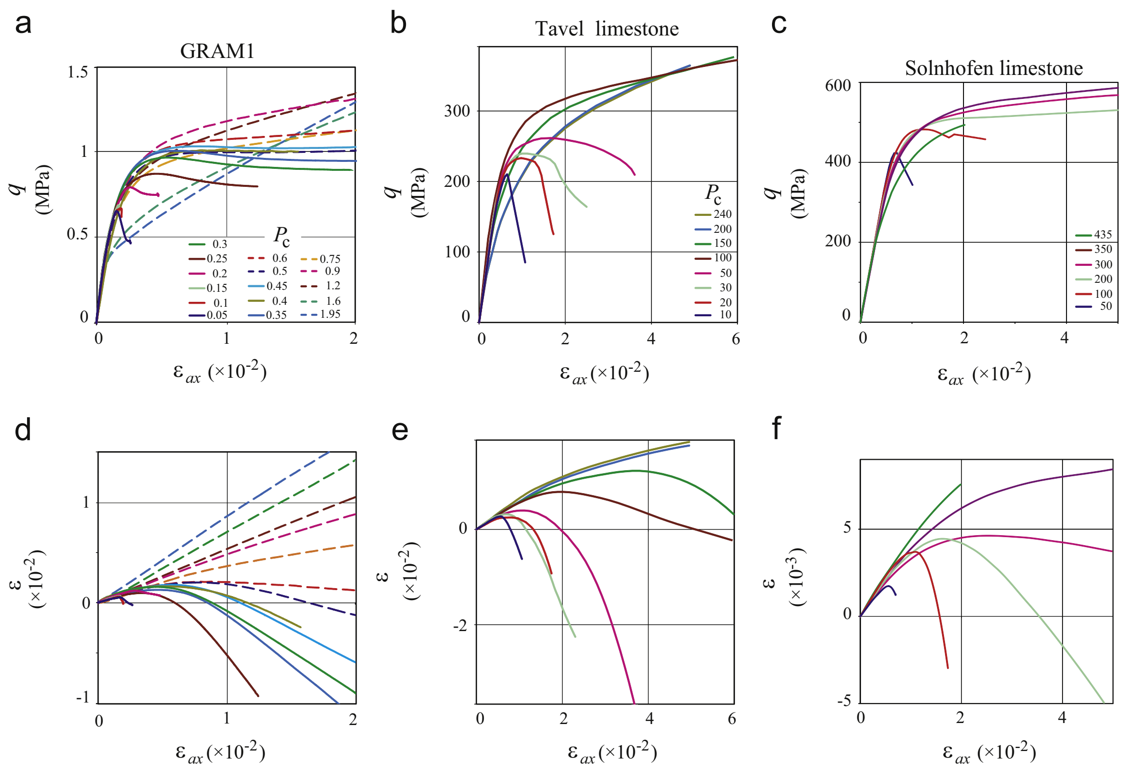

Fig. 1. Mechanical data from the triaxial conventional compression tests for the three materials. $q=\sigma_{ax}-P_c$ is the differential stress, ε_{ax} is the axial strain, and ε is the volume strain (σ_{ax} is axial stress and P_c is the confining pressure). The confining pressure for different curves (tests) is in MPa. (For GRAM1 only one curve for a given P_c is shown, while more curves were used in the data processing). The data for GRAM1 is from [35,38], the TL data is from [36], and the SL data is from [37].

keep the track of evolution of F and G with inelastic deformation, the inelastic strains or their scalar combinations are usually used. In this work we use the inelastic equivalent shear strain $\overline{\gamma}^p$. Defining functions $F(\sigma_{ij},\overline{\gamma}^p)$ and $G(\sigma_{ij},\overline{\gamma}^p)$ represents a real challenge. They are generally poorly constrained even for $\overline{\gamma}^p=0$ (the initial yield and plastic potential functions). Therefore in the applications, the evolution of the constitutive properties (functions F and G) with $\overline{\gamma}^p$ is typically ignored, while it is known from the theoretical analysis [25–29] and numerical simulations [30–34] that this evolution strongly impacts the material deformation and failure.

In the present work we define the constitutive functions using a large data set (31 tests) for Granular Rock Analog Material GRAM1 (described in detail in [35]) as well as the data for Tavel limestone (TL), 8 tests [36] and Solnhofen limestone (SL), 6 tests [37]. All data are from compression conventional triaxial tests conducted for a wide range of confining pressures covering the material behavior from brittle fracturing to ductile flow. These data are shown in Fig. 1 and were processed in our previous paper [38] focused on the definition of the volume inelastic strain $\varepsilon^p(\sigma_m,\overline{\gamma}^p)$ and dilatancy factor $\beta(\sigma_{ij},\overline{\gamma}^p)$. In the present paper we define/map the yield functions $F(\sigma_{ij},\overline{\gamma}^p)$ and the internal friction coefficient $\alpha(\sigma_{ij},\overline{\gamma}^p)$. As expected, the plastic behavior is non-associated since $\alpha(\sigma_{ij},\overline{\gamma}^p)$ and $\beta(\sigma_{ij},\overline{\gamma}^p)$ do not coincide. However, these functions are shown to evolve in a very similar manner with the stress state and with the inelastic strain, which allows to relate F and G and to propose a coherent constitutive formulation based on the experimental data that may be applied at both macro and meso-scales.

2. Theoretical framework

Since σ_{ij} is completely defined by its three invariants for isotropic materials, the constitutive functions for such materials can be expressed in terms of these invariants, the mean stress σ_m (the 1st invariant), the von Mises' equivalent stress $\overline{\tau}$ (the 2nd invariant of the stress deviator tensor that can be expressed through the 1st and 2nd invariants of the stress tensor), and the 3rd invariant that can be taken as the Lode angle which is fixed in our case. Therefore the constitutive functions depend only on σ_m, $\overline{\tau}$, and $\overline{\gamma}^p$. Considering that experimental data suggest deviatoric associativity [3], these functions can be given in the form

$$F=\overline{\tau}-P(\sigma_m,\overline{\gamma}^p) \tag{1}$$

$$G=\overline{\tau}-Q(\sigma_m,\overline{\gamma}^p), \tag{2}$$

where P and G are functions of σ_m and $\overline{\gamma}^p$. What is needed for both theoretical analysis and numerical modeling is not function G itself, but its derivatives $g_{ij}=\partial G/\partial\sigma_{ij}$ defining the direction of the increments of inelastic strain ε_{ij}^p

$$d\varepsilon_{ij}^p=\lambda g_{ij} \tag{3}$$

where λ is a non-negative scalar. Substituting Eq. (2) into Eq. (3) yields

$$d\varepsilon_{ij}^p=\lambda\left(\frac{s_{ij}}{2\overline{\tau}}-\frac{1}{3}Q_{\sigma_m}\delta_{ij}\right) \tag{4}$$

where $Q_{\sigma_m}(\sigma_m,\overline{\gamma}^p)=\partial Q(\sigma_m,\overline{\gamma}^p)/\partial\sigma_m$, and δ_{ij} is the Kronecker delta. Calculating from Eq. (4) the increments of $\overline{\gamma}^p$, $d\overline{\gamma}^p=(2de_{ij}^p de_{ij}^p)^{1/2}$, yields $d\overline{\gamma}^p=\lambda$ and

$$d\varepsilon^p(\sigma_m,\overline{\gamma}^p)=-Q_{\sigma_m}(\sigma_m,\overline{\gamma}^p)d\overline{\gamma}^p \tag{5}$$

where $e_{ij}^p=\varepsilon_{ij}^p-\varepsilon^p\delta_{ij}/3$ and $\varepsilon^p=\varepsilon_{ij}^p\delta_{ij}$. The coefficient (function) relating increments of volume and shear inelastic strains is the dilatancy factor

$$\beta(\sigma_m,\overline{\gamma}^p)=-\frac{\partial\varepsilon^p(\sigma_m,\overline{\gamma}^p)}{\partial\overline{\gamma}^p}=Q_{\sigma_m}(\sigma_m,\overline{\gamma}^p), \tag{6}$$