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Modeling injection/extraction in a fracture network with mechanically interacting fractures using an efficient displacement discontinuity method

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ABSTRACT

The displacement discontinuity method (DDM) is frequently used for modeling the behavior of fractures in reservoir modeling. However, the DDM is not computational efficient for large systems of cracks, limiting its application to small-scale situations. Recent fast summation techniques such as the Fast Multipole (FM) Method accelerate the solution of large fracture problems, demanding linear complexity $O(N)$ in memory and execution time with very modest computational resources. In this work we use the FM-DDM method to simulate fracture response while considering fluid flow through the fracture network. This is a novel and efficient approach for solution of large scale coupled fluid flow-geomechanical problem in naturally fractured reservoirs. Several case studies involving fracture networks with several hundred thousands of boundary elements are presented. The results show a good level of accuracy and computational efficiency compared to the conventional DDM. In addition, the approach is shown to be very useful for the design of exploitation strategies in large-scale fracture network situations. The relative positions between injectors and producers and the fracture permeability variation with injection/extraction play an important role on the distribution of stresses in the fracture network, which in-turn, influence the conditions for the fluid-flow such as fluid pressure and fracture permeability.

1. Introduction

Transient numerical simulations for the coupled fluid flow-geomechanical problem using large number of fracture elements are necessary for realistic representations of unconventional reservoirs for proper design of exploitation strategies. The displacement discontinuity method (DDM) is frequently used for modeling the behavior of natural fractures in linear-elastic rocks [1]. It has been extensively applied in mining and hydraulic fracturing [2–5] due to its semi-analytical nature and by reducing the dimensionality of the problem. However, DDM requires computing the influences among all elements so the coefficient matrix of the system of equations is dense and nonsymmetrical. This requirement impacts the computational performance of conventional strategies, either direct or iterative, for the solution of the system of equations when large numbers of elements are involved, making DDM computationally intensive. Direct solvers requires $O(N^2)$ memory and $O(N^3)$ execution time to compute the coefficient matrix and solve a system of equations. On the other hand, iterative methods still need $O(N^2)$ memory but reduce to $O(kN^2)$ the number of operations for convergence in k iterations, assuming $k \ll N$ in case of well-conditioned systems. However, large-scale problems with several hundred thousands of unknowns are still beyond the current capability of common personal computers.

Recent fast summation techniques such as the Fast Multipole Method (FMM) can accelerate the solution of large fracture problems, demanding $O(N)$ memory and operations easily accessed from personal computers with modest computational resources [9]. FMM relies on a strategy in which the matrix–vector multiplication is accelerated without forming the coefficient matrix explicitly. This acceleration is carried out by efficiently calculating the interaction between elements using the same DDM discretization but by recursive operations of a quad-tree structure for computation and storage. It permits the solution of larger problems, combining the robustness and accuracy of conventional DDM but with superior performance.

Previous works using DDM and FMM has been limited [6,7] mainly because of lack of mathematical developments for the fundamental solutions of interest as well as high programming complexity. Peirce and Napier [6] were the first to explore this area introducing a spectral version of the FMM. They proposed a potential representation of the fundamental solutions to approximate the normal and shear displacement discontinuities, applying the method to the granular assemblies

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problem using up to 1800 boundary elements. On the other hand, Morris and Blair [7] solved the problem of discontinuities within an elastic solid for simulating a brittle rock fracture with 22,500 fracture elements, approximating the far-field behavior of the fundamental solutions by known decaying kernels. These studies [6,7] performed modifications to the DD kernels in order to use the conventional FMM mathematical developments for two-dimensional potential problems, impacting accuracy and efficiency. More recently, Verde and Ghassemi [8] developed a Fast Multipole Displacement Discontinuity Method (FM-DDM) using a kernel-independent version of the FMM [9] which does not require the implementation of multiple expansions of the underlying kernel to compute geomechanical interactions in naturally fractured reservoirs containing up to 100,000 fractures. In contrast to the cited works which have focused only on evaluating static problems dealing with network response, in this work we consider fluid flow through the fractures, and take into account other relevant geomechanical effects such as non-linear joint deformation. The approach improves upon previous works [3–5] allowing for rapid treatment of large scale coupled fluid flow-geomechanical problem in fractured reservoirs. The solution methodology finds transient changes of fluid pressure and normal and shear displacement discontinuities during injection and production operations accounting for joint deformation. Several case studies involving fracture networks with several hundred thousands of boundary elements are presented to evaluate and compare accuracy and computational efficiency of the approach with the conventional DDM, as well as its potential usefulness for the design of exploitation strategies in large-scale situations.

2. Mathematical formulation

2.1. Geomechanical model (FM-DDM)

FM-DDM was developed to compute geomechanical interactions in large-scale fracture network using both the DDM and FFM, respectively, adopting for the latter a kernel-independent version of the classical FMM [9], where analytic multipole and local expansions are not required and using a preconditioned Generalized Minimal Residual Algorithm (GMRES) to solve iteratively the system of equations. In this model for given normal (n) and shear (s) displacement discontinuities (see Fig. 1) in the fractures, defined as the difference in displacement between the two sides of the segment:

$$D_i = u_i(x_1, 0_-) - u_i(x_1, 0_+) \quad i = n, s \tag{1}$$

the normal and shear tractions caused by geomechanical interactions among all fractures can be expressed mathematically as the sum of near and far-field components:

$$t_i = t_i^{near} + t_i^{far} \quad i = n, s \tag{2}$$

where the near-field interactions are evaluated as the conventional DDM showing a quadratic, $O(N^2)$, computational complexity,

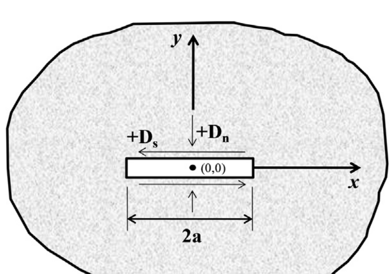

Fig. 1. Fracture segment embedded in a two-dimensional and infinite medium showing constant normal and shear discontinuity displacements [1].

and the far-field influences that involves most of the algebraic products are calculated efficiently using FMM to reduce to $O(N)$ the cost proportional to the number of degrees of freedom (N).

2.1.1. Computation of the near-field interactions by DDM

DDM is an indirect BEM for modeling the normal (opening) and shear (ride) displacement discontinuities of fractures embedded in an infinite and elastic medium [1]. The method is based on fundamental or analytical solutions to the problem of a finite segment fracture centered in a Cartesian plane with constant normal and shear discontinuities in displacement. Using DDM, the tractions at a field point i caused by the effect of m fractures at different source locations can be computed as the sum of the contributions of the m fracture segment involved. If accounting for joint deformation [3–5] and shear dilation [3–5] (normal opening caused by shear displacement), the change in time of the traction components to the normal (Δt_n) and shear (Δt_s) directions can be written as

$$\Delta t_s^i = K_s^i \Delta D_s^i + \sum_{j=1}^{m} A_{ss}^{ij} \Delta D_s^j + \sum_{j=1}^{m} A_{sn}^{ij} \Delta D_n^j = 0 \tag{3}$$

$$\Delta t_n^{i\prime} = K_n^i \left(\Delta D_n^i + \Delta D_s^i \tan \varphi_d \right) + \sum_{j=1}^{m} A_{ns}^{ij} \Delta D_s^j + \sum_{j=1}^{m} A_{nn}^{ij} \Delta D_n^j = \Delta P_k^i \tag{4}$$

where ΔP_k^i represents the change of the fluid pressure inside a fracture element i in a given time step k and the coefficients A_{sn}, A_{ss}, A_{nn}, and A_{ns} represent the global influence containing multiple spatial derivative of the function $f(x,y)$ defined as

$$\begin{aligned} f(x,y) = -\frac{1}{4\pi(1-\upsilon)} \Big[& y \left(tg^{-1}\tfrac{y}{x-a} - tg^{-1}\tfrac{y}{x+a} \right) \\ & -(x-a)\ln\left(\sqrt{(x-a)^2+y^2}\right) + (x+a)\ln\left(\sqrt{(x+a)^2+y^2}\right) \Big] \end{aligned} \tag{5}$$

where f is the relative position function between the element i and j and upon the orientation and length of fracture element j, G is the shear modulus, υ represents Poisson's ratio of the solid medium, and a the fracture half-length.

On the other hand, joint deformation effects are modeled in Eqs. (3) and (4) using the normal (K_n) and shear (K_s) joint stiffness. Based on the Goodman model [10], K_n is estimated via an explicit hyperbolic equation as a function of the initial stiffness (K_{ni}) and maximum closure (D_{nmax}):

$$K_n = K_{ni} \left(1 - \frac{t_n^{i\prime}}{K_{ni} D_{n\,\max} + t_n^{i\prime}} \right)^{-2} \tag{6}$$

where $t_n^{i\prime}$ is the effective normal traction defined as

$$t_n^{i\prime} = \sigma_n^i - P_{k-1}^i \tag{7}$$

with

$$\sigma_n^i = \sigma_{xx}^{\infty} \sin \beta_j^2 - 2\sigma_{xy}^{\infty} \sin \beta_j \cos \beta_j + \sigma_{yy}^{\infty} \cos \beta_j^2 \tag{8}$$

where σ_{xx}^{∞}, σ_{yy}^{∞}, and σ_{xy}^{∞} are the field stress components in xx, yy, and xy directions, respectively.

2.1.2. Computation of the far-field interactions by FMM

2.1.2.1. Brief description of the FMM. The main idea behind the FMM is to accelerate matrix–vector products (Ax) in iterative algorithms without forming the coefficient matrix explicitly, reducing then computation time and saving memory [11,12]. Algebraically, the product of the ith row of a $N \times N$ matrix K with