Instability conditions for systems with distributed time delays via functionals of complete type

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Abstract: Instability conditions for single delay systems of retarded type are given. The approach is based on using the converse results on the existence of special quadratics lower bounds for the Lyapunov Krasovskii functional of complete type associated to these systems.

Keywords: instability, time delay systems, Lyapunov Krasovskii functional of complete type.

1. INTRODUCTION

In the last couple of decades a large amount of literature on stability of delay systems have been reported in the framework of Lyapunov-Krasovskii approach (see Dugard and Verriest (1997), and Gu and Niculescu (2003)), however these functionals are limited, since this is not a constructive approach.

In this sense the approach of Lyapunov-Krasovskii functionals for time delay systems with prescribed derivative has the advantage that the existence of the functional is guaranteed when the system is exponentially stable. The construction of Lyapunov-Krasovskii functionals with prescribed derivative have been addressed in several works such as Repin (1966), Datko (1972), Infante and Castelan (1978), Huang (1989) and Louisell (1998), this problem was revisited and completed in Kharitonov and Zhabko (2003). There, the dependence of the functional on the analogue of the Lyapunov matrix of the system was clarified, and the functional was completed in such a way that it is possible to show that it admits a quadratic lower bound that depends on the Lyapunov matrix. However the construction methods of the analogue of the Lyapunov matrix proposed in Kharitonov and García-Lozano (2002) are possible whether the system is stable or not. As a consequence these functionals have been used mainly in the robustness analysis of time delay systems where the nominal system is known to be exponentially stable.

In Ochoa and Mondié (2007) it is shown how these recent results can be exploited in order to provide instability conditions. The aim of our contribution is to extend these results to systems with distributed time delays based on the negation of the fact that "if the system is stable then it admits a quadratic lower bound". Such a result completes nicely the existing sufficient stability condition based on Lyapunov-Krasovskii functionals of predetermined form: indeed, it is well known that with these results, usually stated in terms of linear matrix inequalities, if the conditions are not satisfied, no conclusion on the stability or instability of the system can be made. In these cases, it is clearly natural to suspect that the system may be unstable. The availability of instability sufficient condition can then be useful for reaching a conclusion.

The contribution is organized as follows: the main results on functionals of complete type are recalled in Section 2 and the existence of a special quadratic lower bound is established in Section 3. The negation of this result leads to the instability conditions presented in Section 4. A number of illustrative examples are given in Section 5 and the contribution ends with some concluding remarks.

2. PREVIOUS RESULTS

Consider linear time delay systems of the form

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h) + \int_{-\sigma}^{0} D(\theta) x(t+\theta) d\theta, \quad t \geq 0 \qquad (1)$$

where $A_0, A_1 \in R^{n\times n}$, $D(\theta) \in R^{n\times n}$ is defined for $\theta \in [-H, 0]$, $H = \max\{\sigma, h\}$, furthermore each element of matrix $D(\theta)$ is continuous and bounded. We denote by $x(t,\varphi)$ the solution of the system with a given initial piece-wise continuous vector function $\varphi(\theta)$, $\theta \in [-h, 0]$. By $x_t(\varphi)$ we denote the segment $x(t+\theta,\varphi)$, $\theta \in [-h, 0]$ of the solution. The Cauchy formula for the solutions of system (1) is

$$x(t,\varphi) = K(t)\varphi(0) + \int_{-h}^{0} K(t-h-\theta) A_1 \varphi(\zeta) d\zeta + \int_{-\sigma}^{0}\int_{\theta}^{0} K(t-\zeta+\theta) D(\theta) d\theta \varphi(\zeta) d\zeta,$$

where $K(t)$ denotes the analogue of the fundamental matrix of the system. It satisfies

$$\dot{K}(t) = A_0 K(t) + A_1 K(t-h) + \int_{-\sigma}^{0} D(\theta) K(t+\theta) d\theta, \ t \geq 0,$$

with initial conditions

$$K(0) = I, \qquad K(\theta) = 0, \quad \theta \in [-H, 0).$$

As every column $K(t)$ is a solution of system (1) it follows that if the system is exponentially stable then, for every constant $n \times n$ matrix W, the matrix

$$U(\tau) = \int_0^{\infty} K^T(t) W K(t+\tau) dt, \quad \forall \tau, \tag{2}$$

is well defined for all $\tau \in R$. The matrix function $U(\tau)$, $\tau \in [-h, 0]$ can be viewed as the analogue of the Lyapunov matrix for the delay system (1). It is shown in Santos et al. (Prague, Czech Rep., 2005) that it satisfies the algebraic property

$$-W = A_0^T U(0) + U(0) A_0 + A_1^T U^T(-h) + U(-h) A_1 + \int_{-\sigma}^{0} U(\theta) D(\theta) d\theta + \int_{-\sigma}^{0} D(\theta) U^T(\theta) d\theta \tag{3}$$

the symmetric property,

$$U(-\tau) = U^T(\tau), \quad \forall \tau, \tag{4}$$

and the dynamic property

$$U'(\tau) = U(\tau) A_0 + U(\tau - h) A_1 + \int_{-\sigma}^{0} U(\tau+\theta) D(\theta) d\theta, \quad \tau \geq 0. \tag{5}$$

Using a nontrivial extension of the stability analysis of linear systems without delay the following results on complete functionals are proved in Santos et al. (Prague, Czech Rep., 2005).

Theorem 1. Santos et al. (Prague, Czech Rep., 2005) Let system (1) be exponentially stable and let $n \times n$ positive definite matrices W_0, W_1, W_2 and M be given. Then the derivative of the functional

$$\begin{aligned}
v(x_t) &= x^T(t) U(0) x(t) \\
&+ 2x^T(t) \int_{-h}^{0} U(-h-\theta) A_1 x(t+\theta) d\theta \\
&+ 2x^T(t) \int_{-\sigma}^{0} \int_{\theta}^{0} U(\theta - \zeta) D(\theta) x(t+\zeta) d\zeta d\theta \\
&+ \int_{-h}^{0} \Big[\int_{-\sigma}^{0} \int_{\zeta}^{0} 2x^T(t+\theta_1) U(\theta_1 - \zeta - \theta_2) D(\theta) d\theta_1 d\zeta \\
&+ \int_{-h}^{0} x^T(t+\theta_1) A_1^T U(\theta_1 - \theta_2) d\theta_1 \Big] \times A_1 x(t+\theta_2) d\theta_2 \\
&+ \int_{-\sigma}^{0} \int_{-\zeta_1}^{0} \int_{-\sigma}^{0} \int_{-\zeta_2}^{0} x^T(t+\theta_1) D(\zeta_1) U(-\zeta_1 + \theta_1 + \zeta_2 - \theta_2) \times \\
&D(\zeta_2) x(t+\theta_2) d\theta_2 d\zeta_2 d\theta_1 d\zeta_1 \\
&+ \int_{-h}^{0} x^T(t+\theta) [W_1 + (h+\theta) W_2] x(t+\theta) d\theta \\
&+ \int_{-\sigma}^{0} (\sigma+\theta) x^T(t+\theta) M x(t+\theta) d\theta,
\end{aligned} \tag{6}$$

satisfies the condition

$$\frac{d}{dt} v(x_t) = -w(x_t). \tag{7}$$

where $w(x_t)$ is

$$\begin{aligned}
w(x_t) &= x^T(t) W_0 x(t) + x^T(t-h) W_1 x(t-h) \\
&+ \int_{-h}^{0} x^T(t+\theta) W_2 x(t+\theta) d\theta \\
&+ \int_{-\sigma}^{0} x^T(t+\theta) M x(t+\theta) d\theta.
\end{aligned} \tag{8}$$

Here, $U(\tau)$, $\tau \in [0, h]$ is defined in (2) and satisfies

$$W = W_0 + W_1 + hW_2 + \sigma M.$$

Remark 1. The functional $v(x_t)$ is fully determined by the matrix $U(\tau)$, $\tau \in [-h, 0]$. Notice that it is not possible to compute $U(\tau)$, $\tau \in [-h, 0]$ using (2). However, it can be constructed by solving the set of equations (4), (5) and (3) as proposed in Kharitonov and García-Lozano (2002).

3. QUADRATIC LOWER BOUND OF THE FUNCTIONAL

The starting point of this contribution is the existence of a quadratic lower bound for the functional (6).

Lemma 2. Let system (1) be exponentially stable and let $n \times n$ positive definite matrices W_0, W_1, W_2 and M be given. Then the functional $v(x_t)$ defined in (6) admits a quadratic lower bound of the form

$$v(x_t) \geq \alpha \left\{ \|x(t)\|^2 + \int_{-h}^{0} \|x(t+\theta)\|^2 d\theta \right\}$$

for any $\alpha \in (0, \alpha_0]$, where $\alpha_0 > 0$ is the first positive value for which the determinants of the matrices

$$L(\alpha) = \begin{pmatrix} W_0 & 0 \\ 0 & W_1 \end{pmatrix} + \alpha \begin{pmatrix} I + A_0 + A_0^T + \sigma I & A_1 \\ A_1^T & -I \end{pmatrix} \tag{9}$$

and

$$M(\alpha) = M + \alpha \delta I \tag{10}$$

vanish for the first time.

Proof. Let us define the functional

$$v^{(\alpha)}(x_t) = v(x_t) - \alpha \left[\|x(t)\|^2 + \int_{-h}^{0} \|x(t+\theta)\|^2 d\theta \right], \tag{11}$$

where $v(x_t)$ is defined in (6). Then

$$\begin{aligned}
\frac{dv^{(\alpha)}(x_t)}{dt} &= -w(x_t) \\
&\quad -2\alpha \left[x^T(t) A_0 x(t) + A_1 x(t-h) \right] \\
&\quad -2\alpha x^T(t) \int_{-\sigma}^{0} D(\theta) x(t+\theta) d\theta \\
&\quad -\alpha \|x(t)\|^2 + \alpha \|x(t-h)\|^2 \\
&= -w^{(\alpha)}(x_t)
\end{aligned}$$

where $w(x_t)$ is defined in (8). Then

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