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Relationship between joint roughness coefficient and fractal dimension of rock fracture surfaces

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ABSTRACT

Numerous empirical equations have been proposed to estimate the joint roughness coefficient (*JRC*) of a rock fracture based on its fractal dimension (*D*). A detailed review is made on these various methods, along with a discussion about their usability and limitations. It is found that great variation exists among the previously proposed equations. This is partially because of the limited number of data points used to derive these equations, and partially because of the inconsistency in the methods for determining *D*. The 10 standard profiles on which most previous equations are based are probably too few for deriving a reliable correlation. Different methods may give different values of *D* for a given profile. The *h–L* method is updated in this study to avoid subjectivity involved in identifying the high-order asperities. The compass-walking, box-counting and the updated *h–L* method are employed to examine a larger population of 112 rock joint profiles. Based on these results, a new set of empirical equations are proposed, which indicate that the fractal dimension estimated from compass-walking and the updated *h–L* method closely relate to *JRC*, whereas the values estimated from box-counting do not relate as closely.

1. Introduction

Discontinuities play an important role in the deformation behavior of a rock mass. Properties of the rock discontinuities include extent, orientation, roughness, infilling and joint wall strength. Roughness, which refers to the local departures from planarity, influences the friction angle, dilatancy and peak shear strength. A milestone was made by Barton [1], who puts forward an empirical equation to estimate the peak shear strength of a rock joint $\tau = \sigma \tan [JRC \log(JCS/\sigma) + \varphi_b]$, where τ is the peak shear strength of the rock joint, σ is the normal stress, *JRC* is the joint roughness coefficient, *JCS* is the strength of joint wall, and φ_b is the basic friction angle. The *JRC* of a particular rock joint profile is most often estimated by visibly comparing it to the 10 standard profiles with *JRC* values ranging from 0 to 20 [2]. This approach was also adopted by the ISRM commission on test methods in 1981 [3]. However, the visual comparison is subjective, since the user has to judge which profile his joint fits the best.

The development of objective methods was gradually advanced by researchers considering statistical parameters and the fractal dimension of the rock joint profiles [4–8]. A detailed review was carried out more recently [9] on the determination of *JRC* using statistical parameters, where empirical equations with R_z (maximum height of the profile), λ (ultimate slope of the profile) and δ (profile elongation index) were proposed and highly recommended for engineering practice as they have high correlation coefficients and are easy to calculate.

The fractal dimension (D) describes the degree of variation in a curve, a surface or a volume from a line, a plane or a cube. Since the work of Turk et al. [10] and Carr and Warriner [11], the fractal dimension was thought to be a suitable parameter for quantifying the roughness of a natural rock joint profile [12–18], as the fractal dimension has a minimum value of 1 for a perfectly smooth profile and a maximum value of less than 2 for an extremely rough undulating profile [19,20]. Numerous empirical equations were put forward for estimating *JRC* using *D*. However, difficulties arise when ranking the suitability of these equations and choosing a particular one to use in engineering practice, as the *D* determination methods, examined profiles and data processing methods on which the empirical equations were based are diverse.

The present study aims to review the determination of *JRC* using *D*. The definition and calculation of *D* determination methods are clearly described, followed by a detailed review of the empirical equations in the literature. The authors will repeat what the previous researchers have done to evaluate the accuracy and limitations of these equations. Finally, 112 joint profiles are utilized

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to correct and update the empirical equations for them to be better used in rock engineering.

2. Fractal dimension and its determination

To date, the fractal dimension of a rock joint profile was generally determined by compass-walking [7], box-counting [24] and the *h–L* methods [25] in rock engineering. A review of these methods is given in the following subsections in terms of definition and calculation.

2.1. Compass-walking method

The compass-walking is also called divider, a yardstick or stick-measuring method [7,21,22], and the main concept of this method is to measure a curve by "walking a compass of radius *r*" along the curve (Fig. 1). The detailed process of measurement is as follows (Fig. 1): set a compass to a prescribed radius *r*, and walk the compass along the profile, each new step starting where the previous step leaves off. For each compass of a certain radius *r*, one would get an *N* (the number of steps) for fully measuring the curve. With compasses of different radii, a set of *N*s would be obtained. If the base 10 log of the *N* values are plotted against the base 10 log of the corresponding *r* values, the slope of this plot is $-D$ [23]:

$$-D=\Delta \log N/\Delta \log r \tag{1}$$

where $\Delta\log N$ is the increment of $\log N$, and $\Delta\log r$ is the increment of $\log r$.

An alternative to the above calculation was used by Maerz et al. [7]. They counted the number *N* of dividers of length *r* needed to cover the profile and repeated this measurements for various lengths of *r*. The fractal dimension *D* is calculated in practice by plotting *Nr* versus *r* in a log–log space and equating the slope to $1-D$:

$$1-D=\Delta \log(Nr)/\Delta \log r \tag{2}$$

where $\Delta\log(Nr)$ is the increment of $\log(Nr)$ in the plot.

A modification of the traditional calculation (1) was made by Bae et al. [21]. The fractal dimension of a joint profile is defined by three parameters including *N*, *r*, and *f*, where, *N* is the number of steps for walking through a joint profile by a divider with a span of *r* (Fig. 1). The length of the joint profile was defined as $Nr+f$, where, the value *f* is obtained by measuring the remaining length shorter than *r* after excluding the length of *Nr* for the total joint profile length. The fractal dimension *D*, thus, is defined as the slope of $\log(N+f/r)$ versus $\log(r)$ according to:

$$-D=\Delta \log[N+(f/r)]/\Delta \log r \tag{3}$$

2.2. Box-counting method

The box-counting dimension is also known as the Minkowski–Bouligand dimension, which works as a way of determining the fractal dimension of a set in a Euclidean space, or more generally in a metric space (*X*, *d*) [24].

To calculate the fractal dimension, the joint profile is placed on an evenly-spaced grid, and the number of boxes required to fully cover the profile is counted. Suppose that *G* is the number of boxes of side length ε required to cover the profile. In practice the box-counting dimension is calculated by seeing how this number changes as the grid gets finer and is obtained by plotting *G*s against the corresponding εs in a log–log space. The slope of this plot is regarded as $-D$:

$$-D=\Delta \log G/\Delta \log \varepsilon \tag{4}$$

2.3. The *h–L* method

This method was firstly proposed by Xie and Pariseau [25], and was defined as:

$$D=\frac{\log 4}{\log\{2(1+\cos[\arctan(2h/L)])\}}$$
$$h=\frac{1}{M}\sum_{i=1}^{M}h_i,\quad L=\frac{1}{M}\sum_{i=1}^{M}L_i \tag{5}$$

where *L* and *h* are the average base length and the average height of "high-order" asperities of a joint, respectively (Fig. 2). A similar definition was also given in the following expression by Askari and Ahmadi [26]:

$$D=\frac{\log 4}{\log\{4\cos[\arctan(2h/L)]\}} \tag{6}$$

The difficulties in using the above two equations are the identification of the so-called "high-order" asperities of a profile and the manual measurement of their base length and height (Fig. 2). The subjectivity involved in identifying the asperities may introduce bias into the estimated *D*.

3. Review of available empirical equations

Since Turk et al. [10], who put forward the first correlation between *JRC* and *D* of a joint profile, studies of this relationship have attracted attention from researchers. Table 1 lists the empirical equations from the literature for estimating *JRC* from *D*; in the text, these equations will be referred to as T1, T2, etc., to avoid confusion with the previous six displayed and numbered equations. It is found that diverse measuring methods for determining *D* were employed, including the compass-walking, box-counting and *h–L* method. Most of the empirical equations (T1, T2, T5, T6, T7, T9, T10, T12, T13 and T17) were derived from the 10 standard *JRC* profiles proposed by Barton and Choubey [2]. Equations (T14-T16) were derived from 10 profiles published by Xu et al. [22]. Equation (T3) was derived from seven profiles published by Qin et al. [27]. Equation (T11) was derived from 42 profiles published by Jia [28]. No clear description of the data source was given for the rest of the equations. The correlation coefficients (if provided) are generally greater than 0.9, showing a close correlation between *JRC* and *D*. Most equations are not accompanied by the sampling interval and the sampling intervals (if provided) are variable.

Equations (T1–T5) take *D* as the independent variable. One of the apparent disadvantages of these equations is that they result in a *JRC* value not equal to 0 for a perfectly smooth plane. That is, they are not applicable for planar or sub-planar joint profiles.

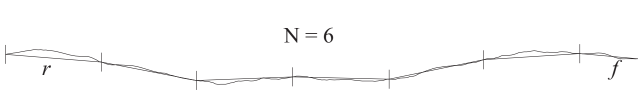

Fig. 1. Schematic of the compass-walking method for determination of *D* of a profile.

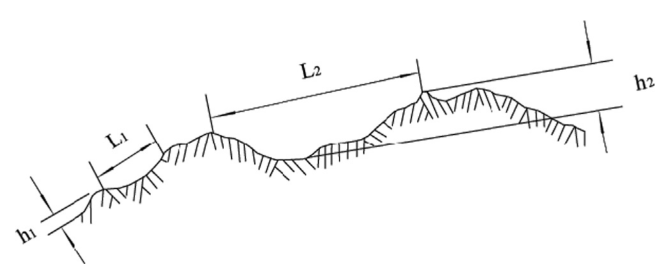

Fig. 2. Measurement of *h* and *L* in Eqs. (5) and (6) (Xie and Pariseau [25]).