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Roughness decomposition and nonlinear fluid flow in a single rock fracture

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A B S T R A C T

The objective of this paper is to investigate the effects of wall surface roughness on fluid flow through rock fractures. A wavelet analysis technique was developed to define a mathematical criterion for decomposing the original wall surface roughness profiles of a fracture into a high-frequency (secondary roughness) profile and a low-frequency (primary roughness) profile, in order to examine their impacts on fluid flow, by solving the Navier–Stokes equations (NSE) without linearization, using a self-developed 2D finite volume method (FVM) code. The results indicate that the high-frequency secondary roughness is the main cause for dynamic evolution of Eddy flow regions in the fracture flow field, besides the Reynolds number (*Re*). In the original fracture model with the high-frequency secondary roughness, our results show that fluid flow fields are not only generally non-linear, but also with non-stop generation and motions of eddies in the boundary layer regions of rough fractures when the $Re = 1000$ in this study, which will affect the solute transport processes in fractured rock masses. The complete NSE were solved without removing acceleration and inertial terms, so that the impacts of surface roughness on the nonlinear and dynamic flow behavior of rock fractures were calculated and visualized more accurately, which is important for modeling mass and energy transport processes in fractures and fractured rock masses.

1. Introduction

1.1. Background and motivation

Crystalline rock masses often contain complex systems of fractures with surfaces of non-stationarity, which provide predominant pathways for fluid flow and solute transport processes. Understanding and quantitative representation of effects of surface geometry complexity on the fluid flow through rough rock fractures therefore are important and challenging issues of energy and mass transport processes in fractured rocks.

In rock mechanics and rock engineering practices, fluid flow in fractures is often and widely assumed to follow the cubic law that is the analytical solution of Reynolds equation derived from linearization of the Navier–Stokes equations (NSE), based on the smooth parallel plate model assumptions [1]. In practice, it is also common to assume local validity of the Local Cubic Law (e.g. [2]) for flow and transport simulations in rough fractures. However, the assumption of the Cubic Law and its local validity may be acceptable only when the flow is largely laminar, and might not be able to provide adequate support for investigating the impacts of surface roughness on energy and mass transport processes in natural fractures, as reported in literature with both laboratory experiments and numerical simulations (e.g. [3–18]).

The first theoretical development for estimating the effects of small scale roughness on transmissivity of rock fractures was reported in Zimmerman et al. [7], by solving the Reynolds equation, a simplified NSE for steady-state flow of incompressible Newtonian fluids though a nominally flat fracture between two symmetrical and slightly non-planar and non-parallel sinusoidal surfaces defined in a 2D space. The results led to analytical expressions for the aperture deviations from the conventional Cubic Law predictions. The model was extended to fracture profiles composed by superposition of two sinusoidal components with different frequencies and amplitudes, and even higher order approximations to NSE. This work was further discussed and verified against experimental data in a comprehensive review by Zimmerman and Bodvarsson [8], which was used as the mathematical basis in Federico [9] and in Sisavath et al. [10] for investigating the non-Newtonian flow through the rough-walled fractures and effects of fracture closure under normal stresses, by a perturbation solution technique. As pointed out in Sisavath et al. [10], the validity of the analytical solution of the Reynolds equation as reported in [7,10] was restricted to long wavelengths and relatively small amplitudes of asperities on fracture surfaces, compared with the mean aperture, and could not predict Eddy

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flow phenomenon, because the nonlinear inertial term was removed from NSE when deriving Reynolds (lubrication) equation.

The issue of inertial term was investigated by Basha and El-Asmar [11] in which a steady state NSE was solved analytically, using perturbation solutions, for idealized rock fracture geometries formed by regularly shaped sinusoidal wavy 2D smooth profiles with different relative positions or two smooth and nonparallel planar or slightly curved profiles. The derived perturbation solutions consist of a leading-order approximation of Reynolds lubrication approximation and higher-order terms representing effects of amplitude and frequency of the asperities and the inertial term.

The above analytical solutions, especially the first time breakthrough reported in [7], provided clear demonstration of the effects of small scale roughness on fluid flow field and corresponding equivalent hydraulic apertures, and played a guiding role for the continued research in the subject area, despite its limitations by simplifications of rough-walled surface geometry of natural fractures. In order to advance our fundamental understanding of dynamic and nonlinear fluid flow in natural rock fractures with more adequate representations of the complex natural surface geometry, experimental studies and directly solving the non-simplified NSE by numerical modeling are needed. This requirement motivated continued research on fluid flow in fractures by laboratory tests and direct numerical solutions of NSE since 1990s, with different techniques and subjects, as briefly discussed below.

In the rock mechanics field, the first integrated experiment in laboratory and numerical modeling for fluid flow in rock fractures was reported in Zimmerman et al. [12] and Al-Yaarubi et al. [13], by testing a Newtonian fluid flow in a sample of replica of a natural fracture and a 3D Finite element solution of the steady state NSE (by removing the acceleration term). It was found that the normalized hydraulic transmissivity started decreasing when $Re > 1.0$, and continued nonlinearly with increasing Re values, due to the combined effect of the fracture surface roughness and inertial nonlinearity. However, no results of roughness analysis or non-laminar flow regions (Eddy) from the numerical modeling were presented or analyzed. A recent integrated laboratory testing and numerical modeling by solving Stokes equation and Reynolds equations was reported in Lee et al. [14], with detailed and quantitative observations, measurements and presentations of the flow field, including Eddy flow regions. The comparison between measured and calculated results indicated important impact of nonlinearity caused by sudden change of surface geometry, when $Re < 0.1$. The reason for overestimation (up to as high as 47–60% at highly rough-walled regions of the tested sample) of flow rate by solving the Reynolds equation was regarded as caused by the abruptly changed geometry during this test. It was also found in [14] that a minor roughness change in the fracture made the Reynolds equation overestimate the flow through the fractures even for $Re < 1$. Unfortunately, no roughness analysis was reported in [14], so that a quantitative correlation analysis between roughness and nonlinear flow was not available.

The accumulated experimental and numerical modeling results demonstrate that the sudden local change of surface geometry is one of the main reasons for Eddy flow generation in rock fractures, as reported in a large number of publications over the years. Cardenas et al. [15–17] solved steady state NSE with 2D and 3D models, with results showing significant effects of surface roughness on the fluid flow and solute transport processes in rock fractures. In order to check effects of roughness quantitatively, in Cardenas et al. [17], four different fracture geometry representations of the same fracture sample were used for fracture geometry model developments, based on a 2D cross-section of a 3D natural rock fracture by X-ray computer tomography (CT) scanning: (1) an actual (i.e. the original) fracture geometry model as scanned; (2) a truncated fracture model with a specially large aperture area near one end of the original model removed; (3) a 'uniform' fracture model generated by duplicating the bottom surface of the fracture with a vertical displacement equal to the mean aperture of the fracture; and (4) a much simplified 'Flat' smooth parallel plate model with the same mean aperture. Such gradual reduction of roughness of the original natural fracture model is effective for observing effects of roughness on Eddy flow generation, but is subjective to the simple means for fracture geometry simplification approach adopted, which cannot provide proper mathematical basis for statistical correlation analysis of the roughness effect on Eddy flow behavior. Crandall et al. [18] examined the relationship between wall-roughness and fluid flow in a series of 3D models of rough-walled tensile fracture by Brazilian testing on a sandstone core, whose 3D volumetric geometry was scanned by using CT, from which six fracture models of varying resolutions of cubical meshes were generated, with no overall non-stationary waviness existed. The roughness of the fracture models were represented by constant values of JRC and fractal parameters, derived from a 2D profile of the tensile fracture. The fluid flow in the six fracture aperture geometry models of different mesh resolutions were modeled by solutions of steady state NSE, using a FVM solver (ANSYS Fluent code). Results from the 6 models shown significant effects of resolutions of surface scanning and numerical model meshing, on hydraulic properties and fluid flow channeling of the fracture sample concerned.

In summary, solving NSE for flow in rough-walled rock fractures is not a new subject in different branches of geosciences, but a subject with two non-resolved outstanding issues: a lack of quantitative analysis of rough surface geometry and its impact on Eddy flow generation and distribution, and a lack of study on non-steady state evolution of Eddy flow regions. The previous efforts were concentrated more on the flow field changes with the Reynolds numbers (Re), often with small Re values, without quantitative analysis of roughness geometry on nonlinear fluid flow behavior, except a conceptual qualitative understanding that roughness is the major cause of Eddy flow. The scanned or computer generated fracture surfaces were mostly directly adopted as geometric boundaries of the problem domains for computational models, without mathematical descriptions of roughness features that can be used for quantitative analysis of roughness and Eddy flow correlations. In addition, to the authors' knowledge, contributions from the acceleration term in the NSE were often ignored. This term may or may not be important when cases with only flow rate with low Re values were concerned, but may need to be considered when different conditions and long-term requirements for energy and mass transport are the main concerns.

The above outstanding issues are the motivations of the research conducted in this paper: (1) development of a rigorous mathematical model for roughness decomposition so that effects of different scales of roughness on generations of Eddy flow regions can be quantified uniquely on a solid mathematical basis; and (2) solving complete NSE, including the acceleration term, so that evolutions of the non-steady state Eddy flow regions can be tested. To the authors' knowledge, research on these two issues was reported for the first time in this paper.

1.2. Importance of roughness characterization

To quantitatively characterize the rock surface roughness, numerous models and parameters were put forward in the past decades, such as joint roughness coefficient (JRC), fractal dimensions and spectral analysis (e.g. [19–21]). A large number of statistical parameters or functions, such as root-mean-square values (RMS), Z2 (related to local slope of profiles), Z3 (related to