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Estimation of the fracture diameter distributions using the maximum entropy principle

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ABSTRACT

Fracture size is often estimated from trace length measurements. It is well recognized to be a typical non-unique inverse problem. This paper presents a distribution-free method for estimating the fracture diameter distributions using moments in conjunction with the maximum entropy principle. In this method, moments are used to characterize the statistical nature of the probability distributions. The method involves the inference of the moments of the true trace lengths from the sampled trace data, the explicit expression of moments of fracture diameters using general stereological relationship, and the estimation of fracture diameter distribution using the maximum entropy principle by a given set of moments of fracture diameters. The proposed method makes it possible to achieve a universal form for the fracture diameter distribution without any particular parametric form, and to match the moments of fracture diameters up to the fourth order. The overall behavior of the method is validated by an example, and the results show that it is capable of estimating the fracture diameter distributions.

1. Introduction

In rock mechanics analysis, fractures are usually encountered and play major roles in controlling the performance of an excavation or engineering structure [1–3]. Stochastic fracture network models are often used for the characterization of rock fractures [4,5]. These methods depend, to a large extent, on the accurate description for the geometry of fractures, including the number, orientation, spacing, location, shape and size, all of which are represented by the probability distributions [6]. However, quantifying the fracture size remains a problem because it's difficult to observe the complete three-dimensional extent straightforwardly. Previous works on estimating the fracture size distributions fall into two categories: "forward modeling" solution [4,7] and "inverse modeling" solution [8]. "Forward modeling" is an approach that the distribution forms of fracture size are assumed a priori and the distribution parameters constantly refined by trial and error to fit the observed trace data. It is a distribution-dependent method, and an appropriate size distribution is therefore conditional on the assumptions being correct. In contrast to the "forward modeling", "inverse modeling" is an approach that the fracture size distributions are determined by the given observed trace data. This approach focuses on stereology because it provides practical techniques for extracting quantitative information about a three-dimensional structure from measurements made on two-dimensional planar sections. Consequently, the characteristics of the three-dimensional fracture size distributions can be inferred from linear samples such as boreholes, or from exposed rock masses such as natural outcrops and excavation surfaces. This process includes two steps: estimation of the true trace length distribution from the measured trace lengths by considering the sampling biases [9–15], and estimation of the fracture size distribution from the derived true trace length distribution [4,9,10,16–18].

While there is a relationship between the trace lengths and the fracture sizes, the estimation of the fracture size distribution by stereology may not necessarily always be robust because of the censorship and sampling bias. In addition, the trace length distribution is insensitive to the choice of the fracture size distribution, which is, in fact, a typical non-unique inverse problem [16–18]. Several approaches have been developed to cope with this issue. One of the approaches is to use the Crofton's theorem. Villaescusa and Brown [16] derived the mean and variance of fracture size for scanline sampling after assuming a specific parametric form for the fracture size distributions (e.g., negative exponential, lognormal), and then checked the assumed fracture size distributions which provided the best match with the

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field trace data. Zhang and Einstein [10] extended Villaescusa and Brown's idea to allow for areal mapping. Lyman [19] explored the applicability of the Crofton's theorem and concluded that the relationship is fortuitously valid for areal mapping, but inapplicable for scanline mapping and should be corrected. The idea for checking the validity of the type of the assumed fracture size distribution by comparing the higher moments of trace lengths with those of fracture size is valid. However, the proposed method has to identify the most probable candidate distribution from among many. Besides, the higher moments of the data are not always match those of the distribution well. Other approaches are the numerical methods and closed-form methods. Song and Lee [17] derived the trace length distribution on an infinite surface for three kinds of fracture traces called contained, dissecting and transecting traces, and then estimated the fracture size distribution from the trace length distribution through discretizing the fracture diameter distribution in Warbuton's equation using a numerical method. Tonon and Chen [18] use the Santalò closed-form integral solution to derive closed-form solutions of the fracture size distribution for common trace length distributions (uniform, exponential, gamma, and power law), as well as numerical solution for the case of a lognormal distribution of trace lengths.

All of these methods require an assumption of the parametric form, however, there is no universally accepted form for the assumed distributions of trace lengths or fracture sizes [18]. A built-in problem is that the classical theoretical distributions and corresponding parameters are largely affected by the statistical inference methods, such as parameter estimation and hypothesis testing methods, which is even more uncertain because the complexity of the geological processes in real applications.

This paper presents a distribution-free method for estimating the fracture diameter distribution, using moments with the maximum entropy principle. In fact, moments are significant quantities to describe the characteristics of random variables. Approximation of a distribution function by another function possessing even just the same four lower moments is often found to be good enough [20]. Using the maximum entropy principle, a universal form for the probability distribution without having to assume any parametric form can be achieved, and moments of the fracture diameter distribution can be matched well with those of the field trace data.

2. Inference of moments of true trace lengths from an irregular convex window

In developing the methodology for estimating fracture size distribution, the Poisson disk model [10,11] is used, in which fractures are planer and thin circular disks, with their centers randomly and independently distributed in space.

Observations of fracture traces at rock exposures are subjected to orientation, size, censoring and truncation biases. Orientation bias occurs because the probability of a fracture appearing in an outcrop depends on the relative orientation between the outcrop and the fracture; size bias occurs because large fracture traces are more likely to be sampled than small ones; censoring bias occurs since trace lengths longer than a fixed distance cannot be observed in sample window; truncation bias occurs when trace lengths below some cut-off length are not recorded [21]. Since it is practically feasible to observe and measure trace lengths as low as 0.1 m both in the field and from photographs [10], the truncation threshold is easily decreased to an adequate low value so that it has negligible influence on the trace length estimates. Therefore, the truncation bias is not considered in this work. The orientation bias, size bias and censoring bias in sampling for trace lengths are considered when inferring the true trace length distribution from the measured trace lengths.

2.1. Moments of trace lengths intersecting an irregular convex sampling window

When fractures intersect a sampling window, the intersection may occur in three ways: (*a*) traces with both ends censored, termed transecting traces; (*b*) traces with one end censored and one end observable, termed dissecting traces; and (*c*) traces with both ends observable, termed contained traces. If the numbers of traces in each of the above three types are N_T, N_D and N_C, respectively, the total number of traces, N, will be

$$N = N_T + N_D + N_C. \tag{1}$$

Consider an irregular convex window with area A of window "width" $w(\theta)$ in the direction θ (Fig. 1): it is convenient to think of the irregular convex window as a sum of rectangles of finite width $w_i(\theta)$ and height h_i in the θ direction (Fig. 2). Thus, fracture traces intersect with the irregular convex window can be calculated by considering the summation of the intersections of traces with each rectangular subdomain of the sampling window.

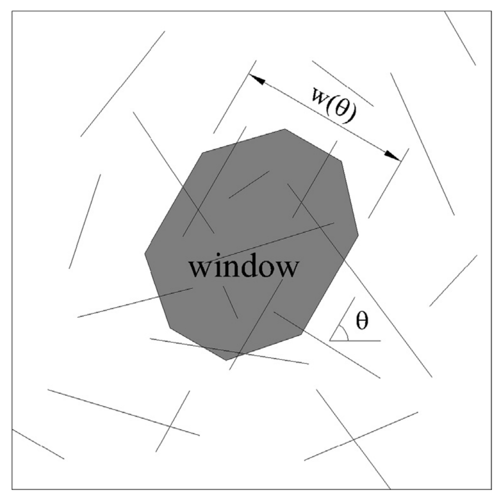

Fig. 1. Randomly oriented fracture traces sampled by an irregular convex window. The "width" of the window in the θ direction is denoted $w(\theta)$.

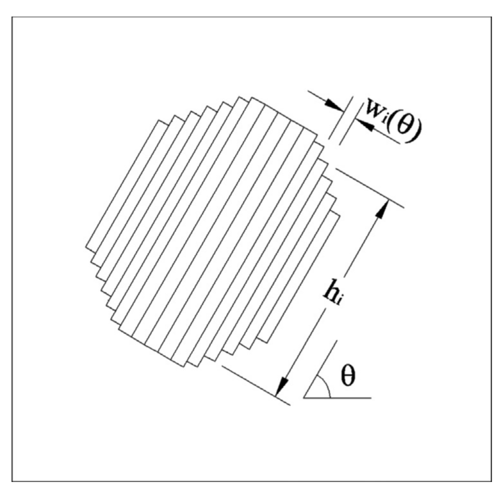

Fig. 2. The irregular convex window is divided into a set of rectangular subdomains of width $w_i(\theta)$ and height h_i in the θ direction, thus, the intersection of traces with the irregular convex window are considered as the summation of the intersections of traces with each rectangular subdomain.