International Journal of Rock Mechanics & Mining Sciences 72 (2014) 277–282

Contents lists available at ScienceDirect

International Journal of Rock Mechanics & Mining Sciences

journal homepage: www.elsevier.com/locate/ijrmms

A mechanisms-based model for dynamic behavior and fracture of geomaterials

A. Zubelewicz [a,*,1], E. Rougier [b], M. Ostoja-Starzewski [c], E.E. Knight [b], C. Bradley [b], H.S. Viswanathan [b]

[a] Alek & Research Associates, LLC, Los Alamos, NM, USA
[b] Los Alamos National Laboratory, Los Alamos, NM, USA
[c] Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA

ARTICLE INFO

Article history:
Received 31 January 2014
Received in revised form
27 August 2014
Accepted 26 September 2014

Keywords:
Dynamic fracture
Rate-dependent frictional plasticity
Anisotropy

ABSTRACT

A mechanisms-based fracture model applicable to a broad class of earth and earth-like materials is presented. The key features the model captures are: (1) material anisotropy; (2) rate-sensitive directional fracture; (3) dilatational friction; (4) dynamic overstress in loading extremes, where the rate of supplied energy is not fully compensated by the rate of the energy redistribution and release and, lastly, (5) spatial stochasticity due to material heterogeneity. In comparison with more traditional phenomenological descriptions, the contribution of the proposed approach is the utilization of tensor representation theory; the theory is suitable for converting observed deformation and fracture mechanisms into a precise mathematical description of the material's behavior.

1. Introduction

Despite notable successes, the development of a predictive fracture model for geomaterials still poses a challenge. A broad review of fracture mechanisms in rocks is presented in [1,2]. The most common approach in modeling brittle fracture relies on incorporating damage characteristics into the Gibbs free energy [3–5]. Several constitutive models have been proposed, and among them the microstructural models discussed in [6–8] are worth noting. The microplane model [9,10] is the closest to our approach and is based on the simple idea of tracking fracture at microplanes with pre-determined orientations. In this setting, the fracture planes carry information about the damage process in a physically justifiable manner. Furthermore, an assumption made in the microplane model is that the macro-damages can be treated as weakly interacting cracks. Since the crack opening strain is collected from the individual micro-cracks, this description is suitable for predicting the material behavior at early stages of the damage process. The cracks are characterized in terms of the crack surface area, while their orientations define the relevant stress tractions. In other approaches, viscous-like stress intensity factors are incorporated into the material description [11]. The time dependence is shown to replicate observed rock strengthening in the Hopkinson Bar strain rate regime (10^3/s to 10^4/s). However, it is not clear whether the relationship holds at extreme strain rates (shock conditions).

The proposed approach utilizes results obtained in [12,13], where the tensor representation theory is shown to be a very useful tool in the hands of a modeler [14]. First, we determine the dominant deformation and fracture mechanisms in a geomaterial. The selected mechanisms are incorporated into the mechanisms-based visco-plasticity model. In geomaterials the material's strength is mildly strain rate dependent until the point when the rate of supplied energy becomes comparable with the rate of the energy redistribution and release due to cracking. At extreme conditions the balance is violated and the material has an excess of energy. We study the mechanism by which, instead of a single dominant crack, the dynamically overstressed heterogeneous material experiences complex fractures and, in this manner, efficiently converts the externally supplied energy into the crack surface formation energy.

2. Tensor representations

In the proposed fracture model, fracture processes are described in terms of dyadic products $N_{ij} = n_i n_j$, where unit vector n_k defines the crack orientation. We want the tensor to be expressed in terms of another symmetric tensor (for instance, stress). In here, we choose the base tensor to be $T_{ki} = T_{ik}$ and, then,

* Corresponding author.
E-mail address: Alek.Zubelewicz@gmail.com (A. Zubelewicz).
[1] Formerly at Los Alamos National Laboratory, Los Alamos, NM, USA.

http://dx.doi.org/10.1016/j.ijrmms.2014.09.015
1365-1609/

express N_{ij} in terms of T_{kl} such that $N_{ij}^T = N_{ij}^T(T_{kl})$. The tensor representation theory [14] says that any symmetric second order tensor can be represented by another symmetric tensor, if the original tensor and its representation have the same first, second and third invariants. From the Cayley–Hamilton theorem we write

$$N_{ij}^T = \alpha_0\delta_{ij} + \alpha_1 T_{ij}^D + \alpha_2 T_{ik}^D T_{kj}^D, \tag{1}$$

where the deviator of T_{kl} is $T_{kl}^D = T_{kl} - T_{ii}\delta_{kl}/3$. There are three representations of the dyadic product each aligned with a principle orientation of T_{kl}. First, we notice that the tensor representation N_{ij}^T taken to the first, second, and third power is still the same tensor $N_{ij} = N_{ik}N_{kj} = N_{ik}N_{kl}N_{lj}$ and their traces are equal to one. As a result, we find three solutions in terms of $\{\alpha_0, \alpha_1, \alpha_2\}$ and, consequently, we have three tensor representations. It can be shown that the eigenvector consists of $T_1 = N_{ij}^1(\alpha_0^1, \alpha_1^1\alpha_2^1)T_{ij}$, $T_2 = N_{ij}^2(\alpha_0^2, \alpha_1^2\alpha_2^2)T_{ij}$ and $T_3 = N_{ij}^3(\alpha_0^3, \alpha_1^3\alpha_2^3)T_{ij}$. For instance, when tensor T_{kl} is stress then the eigenvector consists of principal stresses $\{\sigma_1, \sigma_2, \sigma_3\}$, where $\sigma_1 > \sigma_2 > \sigma_3$. The first set of the alpha parameters is

$$\begin{aligned} \alpha_0^1 &= \frac{1}{3}\left(1 - 2\cos\frac{\pi+\varphi}{3}\sec\varphi\right) \\ \alpha_1^1 &= \frac{8\sqrt{3}\cos\frac{\varphi}{3} - 2\sqrt{3}\cos\varphi - 3\sqrt{3}\sec\varphi - 6\sin\varphi}{6\sqrt{J_2}\left(2\cos\frac{2\varphi}{3} - 1\right)\left(\sqrt{3} - 2\sin\frac{2\varphi}{3}\right)} \\ \alpha_2^1 &= \frac{\cos\frac{\pi+\varphi}{3}\sec\varphi}{J_2} \end{aligned} \tag{2}$$

The second representation is described by

$$\begin{aligned} \alpha_0^2 &= \frac{1}{3}\left(1 + 2\cos\frac{\pi+\varphi}{3}\sec\varphi\right) \\ \alpha_1^2 &= \frac{2\tan\varphi\cos^2\frac{\varphi}{3}\sec\varphi}{\sqrt{3J_2}\left(1 + 2\cos\frac{\varphi}{3}\sec\varphi\right)} \\ \alpha_2^2 &= -\frac{\cos\frac{\varphi}{3}\sec\varphi}{J_2} \end{aligned} \tag{3}$$

and the third one becomes

$$\begin{aligned} \alpha_0^3 &= \frac{1}{3}\left(1 - 2\cos\frac{\pi-\varphi}{3}\sec\varphi\right) \\ \alpha_1^3 &= \frac{8\sqrt{3}\cos\frac{\varphi}{3} + 2\sqrt{3}\cos\varphi - 3\sqrt{3}\sec\varphi - 6\sin\varphi}{6\sqrt{J_2}\left(2\cos\frac{2\varphi}{3} - 1\right)\left(\sqrt{3} - 2\sin\frac{2\varphi}{3}\right)}. \\ \alpha_2^3 &= \frac{\cos\frac{\pi-\varphi}{3}\sec\varphi}{J_2} \end{aligned} \tag{4}$$

In all the cases, the angle is $\varphi = \sin^{-1} J_3(27/4J_2^3)^{1/2}$, while the second and third invariants are $J_2 = T_{ij}^D T_{ij}^D/2$ and $J_3 = T_{ik}^D T_{kl}^D T_{li}^D/3$, respectively.

3. Friction-induced visco-plasticity

Frictional plastic deformation consists of slippages along crack surfaces and there is an out-of-plane dilatation due to crack roughness. The shear strain rate is defined in terms of the Mises–Schleicher concept developed independently by von Mises and Schleicher [15]. In our case, the mechanism of the frictional shear flow is

$$M_{ij} = \sqrt{3}\,\frac{S_{ij}}{\sqrt{J_2^\sigma}} + q(J_2^\sigma)\delta_{ij} \tag{5}$$

such that

$$\dot{\varepsilon}_{ij}^p = \frac{1}{2}M_{ij}\dot{e}_{eq}^p. \tag{6}$$

The flow tensor M_{ij} is co-rotational with stress deviator $S_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$ and is a function of the second invariant of stress deviator $J_2^\sigma = S_{ij}S_{ij}/2$. Also, it includes the dilatation part scaled through internal friction $q = q_0/[1 + (\sqrt{3J_2^\sigma}/\sigma_q)^{n_q}]$, where q_0 is friction parameter. Friction is affected by shear (second invariant of stress deviator), where σ_q and n_q are constants. The equivalent plastic strain rate $\dot{e}_{eq}^p$ is coupled with an equivalent stress. From the requirement of measure invariance (independence from the frame of description) of plastic power $(\sigma_{ij}\dot{\varepsilon}_{ij}^p = \sigma_{eq}\dot{e}_{eq}^p)$, the equivalent stress becomes $\sigma_{eq}^p = M_{ij}\sigma_{ij}/2 = \sqrt{3J_2^\sigma} + 3pq/2$. In the next step, we develop a constitutive relationship between the equivalent stress and strain rate. In here, the relationship is proposed in a pseudo-linear form

$$\dot{e}_{eq}^p = \dot{\Lambda}_p \frac{\sigma_{eq}^p}{\sigma_0^p} \tag{7}$$

where σ_0^p is the material's strength consisting of static strength σ_0^S and dynamic overstress σ_0^D such that $\sigma_0^p = (1-\eta_c)(\sigma_0^S + \sigma_0^D)$. The role of the two strengths and damage (η_c) are discussed later. We correct the linear strain rate dependence given by $(\sigma_{eq}^p/\sigma_0^p)$ and make it non-linear by using a rate-sensitive factor $\dot{\Lambda}_p = \dot{e}_N^0\left(\dot{e}_N^t/\dot{e}_N^0\right)^{\omega_p}$, where the parameter ω_p is a material constant. Lastly, we choose the scaling parameter $\dot{e}_N^0$ to be equal to $1/s$. The normalized total strain rate is $\dot{e}_N^t = \sqrt{\dot{\varepsilon}_{ij}^t\dot{\varepsilon}_{ij}^t/2}$. In this construction, we allow the effective stress exponent to vary throughout the deformation process. At an advanced stage of the plastic deformation, where $\dot{\varepsilon}_{ij}^p \to \dot{\varepsilon}_{ij}^t$ we have $\dot{e}_{eq}^p \propto (\sigma_{eq}^p/\sigma_0^p)^{1/(1-\omega_p)}$; thereby assuring a smooth elasto-plastic transition. As geomaterials generally exhibit low strain rate dependence, the parameter ω_p is expected to have values only slightly smaller than one.

4. Fracture tensor

In geomaterials, the fracture processes occur in tension, compression and shear. Multiple tensile micro-cracks form on microplanes and are predominantly aligned with the direction of maximum tensile stress. The extent of the damage is correlated with the amount of work required for the micro-cracks to become fully opened [16]. In order to monitor the damage process, we construct a fracture tensor, whose rate is defined as follows

$$\dot{\Omega}_{ij} = \dot{G}_f N_{ij}^\sigma, \tag{8}$$

where the tensor $N_{ij}^\sigma = n_i n_j$ is a dyadic product of vector n_i aligned with the direction of maximum tensile stress. In Section 2, we have expressions for the three representations $N_{ij}^\sigma(\sigma_{kl})$ and in here we take the first one defined by parameters $\{\alpha_0^1, \alpha_1^1, \alpha_2^1\}$. Fracture energy is equal to the work needed for opening the cracks, thus $\dot{G}_f = \left(N_{ij}^\sigma\sigma_{ij}\right) \times \left(l_c N_{kl}^\sigma \dot{\varepsilon}_{kl}^e\right)$. The normal component of traction vector is $N_{ij}^\sigma\sigma_{ij}$ and the rate of crack opening displacements becomes $l_c N_{kl}^\sigma \dot{\varepsilon}_{kl}^e$. Lastly, the characteristic fracture length l_c is correlated