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Technical Note

A new approach to obtain rheological relations for saturated porous media

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1. Introduction

Hydrogeomechanical models are important for solving various problems in hydrogeology, hydrogeoecology, oil production and geophysics. The main modern concepts of hydrogeomechanics are presented, for example in Refs. [1,2]. Oil depletion in some Russian oil fields necessitates formulating models that take into account the variation of the stress–strain state of the rock mass caused by chemical interactions between components of underground fluid and the material of the porous skeleton, in order to perform effective enhanced oil recovery. Those models are also essential in problems of hydrogeology, such as filtration of solutions in clay layers, suffusion processes and karst processes.

The above-mentioned chemical interactions usually cause the variation of the mass of the porous matrix [3]. That is why it is important to perform additional research into the influence of this variation on rheological relations, which are required to obtain a closed model of deformations of filtrating porous media. It is also necessary to perform systematic development of main equations of underground mass-transfer in this case. Whlie those questions did not receive exhaustive explanation in specialized literature, it makes sense to obtain required equations and examine most important applications.

2. Mass balance equations of porous skeleton and percolating liquid

First, it is essential to develop a set of equations of filtration in a deformable porous medium with porous and variable-mass skeleton. From the definition of the volume strain of the porous medium θ:

$$\theta = (V - V_0)/V_0. \tag{1}$$

Assuming values of θ are small, we can obtain the following expression:

$$V = V_0 \exp\theta \tag{2}$$

where V is the representative volume of the porous medium, and the subscript "0" stands for initial values at zero time. Therefore, for the mass of the porous medium we have

$$M_s = (1-m)\rho_s V_0 \exp\theta \tag{3}$$

where ρ_s is the density of the solid phase and m is the porosity of the rock. If the last equation is differentiated with respect to time, we find

$$\frac{\partial m}{\partial t} = \frac{(1-m)}{\rho_s}\frac{\partial \rho_s}{\partial t} + (1-m)\frac{\partial \theta}{\partial t} - \frac{(1-m)}{M_s}\frac{\partial M_s}{\partial t}. \tag{4}$$

The mass balance of the solid material of the porous skeleton is described by the equation

$$\partial[(1-m)\rho_s]/\partial t + \mathrm{div}[(1-m)\rho_s \mathbf{W}] = j \tag{5}$$

where $\mathbf{W}$ is the velocity of the solid phase, and j denotes the source/sink of the mass of the porous skeleton caused by interface interaction. The porous skeleton is assumed to lose its mass during interface interaction; so, hereinafter j will represent the discharge

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[1] http://kpfu.ru/eng
[2] http://www.jscc.ru/eng/index.shtml

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of the mass. The mass of the material of the porous skeleton in the representative volume there can be defined as

$$\rho_s V_s = \rho_s(1-m)V = M_s \tag{6}$$

where V_s is the volume of the solid phase in the representative volume. Discharge variable in Eq. (5) is the loss of the mass of the porous skeleton caused by processes like dissolution, leaching or suffusion. It can be written as

$$j = \frac{1}{V}\frac{\partial M_s}{\partial t}. \tag{7}$$

From Eq. (5), after expanding the derivatives, we have

$$-\frac{\partial m}{\partial t}\rho_s + (1-m)\frac{\partial \rho_s}{\partial t} + (1-m)\rho_s \mathrm{div}\,\mathbf{W} + \mathbf{W}\,\mathrm{grad}[(1-m)\rho_s] = j. \tag{8}$$

Taking into account Eqs. (4) and (7) and assuming the last term of the left part of Eq. (8) to be the second order infinitesimal:

$$\partial\theta/\partial t = \mathrm{div}\,\mathbf{W}. \tag{9}$$

The last assumption is the traditional one in poromechanics [4].

Mass balance of the dissolving fluid in saturated rock is described by

$$\partial(m\rho)/\partial t + \mathrm{div}(m\rho\mathbf{V}) = 0 \tag{10}$$

where ρ is the density of the fluid and V is the velocity of the fluid. The relative velocity of the fluid in rock (filtration velocity) is $\mathbf{q} = ms(\mathbf{V}-\mathbf{W})$. Then, from Eqs. (9) and (10) we have

$$m\partial\rho/\partial t + \rho\partial m/\partial t + \mathrm{div}(\rho\mathbf{q}) + \mathrm{div}(\rho m\mathbf{W}) = 0. \tag{11}$$

After transformation of the last equation, with $\mathbf{q}\,\mathrm{grad}\,\rho$ and $\mathbf{W}\,\mathrm{grad}(m\rho)$ considered to be infinitesimal values, we find

$$m\rho^{-1}\partial\rho/\partial t + \partial m/\partial t + \mathrm{div}\,\mathbf{q} + m\partial\theta/\partial t = 0. \tag{12}$$

Combined with Eq. (4), Eq. (12) gives the following expression:

$$\left[m\rho^{-1}\frac{\partial\rho}{\partial t} + (1-m)\rho_s^{-1}\frac{\partial\rho_s}{\partial t}\right] + \frac{\partial\theta}{\partial t} + \mathrm{div}\,\mathbf{q} = \frac{(1-m)}{M_s}\frac{\partial M_s}{\partial t}. \tag{13}$$

It is obvious from $\rho_s = M_s/V_s$ that

$$(1-m)\rho_s^{-1}\frac{\partial\rho_s}{\partial t} = \frac{(1-m)V_s}{M_s}\frac{\partial(M_s/V_s)}{\partial t}. \tag{14}$$

Expanding the derivative in the right side and taking Eq. (13) into account gives

$$(1-m)\rho^{-1}\frac{\partial\rho}{\partial t} + \frac{\partial\theta}{\partial t} + \mathrm{div}\,\mathbf{q} = \frac{(1-m)}{V_s}\frac{\partial V_s}{\partial t}. \tag{15}$$

The right side of Eq. (15) can be expressed as

$$\frac{(1-m)}{V_s}\frac{\partial V_s}{\partial t} = (1-m)\frac{\partial\varepsilon}{\partial t} \tag{16}$$

where ε is the volume strain of the skeleton material (sum of the diagonal elements of the strain tensor). Then, Eq. (15) can be written as

$$(1-m)\rho^{-1}\frac{\partial\rho}{\partial t} + \frac{\partial\theta}{\partial t} + \mathrm{div}\,\mathbf{q} = (1-m)\frac{\partial\varepsilon}{\partial t}. \tag{17}$$

Water is a slightly compressible fluid; so, the first term in the left side of Eq. (17) can be neglected, yielding

$$\frac{\partial\theta}{\partial t} + \mathrm{div}\,\mathbf{q} = (1-m)\frac{\partial\varepsilon}{\partial t}. \tag{18}$$

Time integration of the last equation gives

$$\theta + \int_0^t \mathrm{div}\,\mathbf{q}\,d\tau = \int_0^t (1-m)\frac{\partial\varepsilon}{\partial t}d\tau. \tag{19}$$

Partial integration of the right side leads to the equation

$$\theta + \int_0^t \mathrm{div}\,\mathbf{q}\,d\tau = (1-m)\varepsilon + \int_0^t \varepsilon\frac{\partial m}{\partial t}\,d\tau. \tag{20}$$

Using Eq. (4) for the time derivative of the porosity and Eq. (14) for the term $(1-m)\rho_s^{-1}\partial\rho_s/\partial t$ we can transform Eq. (20) to the following equation:

$$\theta + \int_0^t \mathrm{div}\,\mathbf{q}\,d\tau = (1-m)\varepsilon + \int_0^t \varepsilon\left[(1-m)\frac{\partial\theta}{\partial t} - (1-m)\frac{\partial\varepsilon}{\partial t}\right]d\tau. \tag{21}$$

It is obvious that $\varepsilon \le \theta$; so, for the last term of the right side of Eq. (21) it is true that

$$\int_0^t \varepsilon\left[(1-m)\frac{\partial\theta}{\partial t} - (1-m)\frac{\partial\varepsilon}{\partial t}\right]d\tau \le \int_0^t \left[(1-m)\,\theta\frac{\partial\theta}{\partial t} - (1-m)\varepsilon\frac{\partial\varepsilon}{\partial t}\right]d\tau. \tag{22}$$

This last integral contains time derivatives of the squares of ε and θ, so it can be neglected. Therefore, finally we arrive at

$$\theta + \int_0^t \mathrm{div}\mathbf{q}\,d\tau = (1-m)\varepsilon. \tag{23}$$

The obtained equation has clear physical sense: the total strain of the porous rock is made up of two terms. The first term is the water that was forced out during the filtration. The second term is deformation of the porous skeleton.

3. Rheological relations for the filtrating of porous media with mass-variable porous skeleton

Usually rheological relations are obtained from the expression for the free energy of the porous medium [1]. Dissipation caused by chemical reactions is not included in this approach, and that is why it can hardly be applied in our case. In order to obtain rheological relations we consider that the solid skeleton is elastic; then, we have [5,6]

$$\sigma_{ij}^s = -\left(K - \frac{2}{3}G\right)\varepsilon\delta_{ij} - 2G\varepsilon_{ij}, \quad \varepsilon = \sum_i \varepsilon_{ii} \tag{24}$$

where ε_{ij} is the strain tensor of the skeleton and G is the shear modulus. In the one-dimensional case, vertical compressive stress σ_{zz} can be expressed as

$$\sigma_{zz}^s = -\left(K + \frac{4}{3}G\right)\varepsilon. \tag{25}$$

Denoting $\alpha = K + (4/3)y$, we can write (23) as

$$\alpha\left[\theta + \int_0^t \mathrm{div}\,\mathbf{q}\,d\tau\right] = -(1-m)\sigma_{zz}^s. \tag{26}$$

From the definition of actual stresses [5], the right side of Eq. (26) can be written as

$$-(1-m)\sigma_{zz}^s = P - mp. \tag{27}$$

From the definition of the effective stress tensor $\sigma_{ij}^f = (1-m)(\sigma_{ij}^s - p\delta_{ij})$ [4], and from the equation of non-inertial motion of solid and liquid phases,

$$\frac{\partial\sigma_{ij}^f}{\partial x_j} - \frac{\partial p}{\partial x_i} + [m\rho + (1-m)\rho_s]F_i = 0 \tag{28}$$

where F_i is the density of the external mass forces, and from Eq. (27), it follows that

$$\sigma^f = \alpha\theta, \quad \alpha\int_0^t \mathrm{div}\,\mathbf{q}\,d\tau = (1-m)p. \tag{29}$$

The first equation is the rheological relation for the volume strains of the filtrating medium. Shear deformations obviously coincide with deformations of the porous medium, so rheological relations can be written as

$$\sigma_{ij}^f = - \le (K - \tfrac{2}{3}G)\theta\delta_{ij} - 2G\varepsilon_{ij}. \tag{30}$$