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Technical Note

Predicting the stability of hard rock pillars using multinomial logistic regression

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A R T I C L E I N F O

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1. Introduction

Pillars are present in most underground hard rock mines and they are mainly designed to ensure the protection of roadways and entries. Pillars are often located within the orebody and the mine operators have to maximise the extraction of this valuable resource while maintaining the stability of the pillars. Pillars must then be properly designed because failure to do this may result in either pillar failure or over conservative designed pillar.

Pillar stability must be guaranteed through the entire life of the underground mine that can be years or even decades long, although some of them, such as the yielding pillars, may be allowed to fail. The stability can be analysed by a number of methods and they are generally represented by the ratio between the pillar strength and pillar load that is expressed in Factor of Safety (*FoS*). However, due to uncertainties in material properties, non-regular geometries and different mining operations, the *FoS* calculated using a deterministic approach had some intrinsic limitations [1]. Pillars with calculated *FoS* greater than 1 may still fail and on the other hand pillars with calculated *FoS* less than 1 may still be stable.

Design charts for hard rock pillars that are based on the actual stability data can then be considered as alternative design tools and starting points before a more sophisticated engineering analysis is conducted. These charts have been proposed over the years [2–9] where the latest one included all stability compiled previously and suggested "The Confinement Formula" to generate pillar stability charts. Research reported in this paper utilised the database used in Ref. [9] and further analysed the data by implementing the multinomial logistic regression that is basically a similar approach to the binary logistic regression for coal pillars that was given in Ref. [10]. The main difference of these two approaches lies in the stability data. In Ref. [10] the stability data were grouped into two, which were stable and failed, whereas in this research the stability data were extended into stable, unstable, and failed as shown later in this paper.

2. Logistic regression analysis

2.1. Binary logistic regression model

Binary logistic regression is a statistical modelling technique where the dependent variable (Y) has only two possible values and it is a useful tool for analysing data that includes categorical response variables, such as yes/no or live/die or stable/failed, as compared to the regression of numerical values. Binary logistic regression does not model the dependent variable Y directly but it is rather based on the probabilities associated with the values of Y. For simplicity, and because it is the case most commonly encountered in practice, Y can be coded as 1 in the case of positive outcome or success (i.e. yes or live or stable) and coded as 0 in the case of negative outcome (i.e. no or die or failed). If there is a collection of p independent variables denoted by the vector $\boldsymbol{x}' = (x_1, x_2, \ldots, x_p)$, the probability of cases for which $Y = 1$ is defined as [11]

$$P(Y=1|\boldsymbol{x}) = \pi(x) = \frac{e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p}}{1 + e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p}} \quad (1)$$

where β_0, $\beta_1, \ldots, \beta_p$ are the logistic regression model parameters, which can be determined through special methods which have been programmed as an iterative weighted least squares procedure in available logistic regression software [12].

The binary logistic regression has been recently implemented in predicting the stability of coal pillars which was reported in details in Ref. [10].

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2.2. Multinomial logistic regression model

The binary logistic regression model can be extended to handle the case where the dependent variable has more than two possible values [13]. The model is frequently referred to as the discrete choice model in business and econometric literature while it is call the multinomial, polychotomous, or polytomous logistic regression model in the health and life sciences [11].

Furthermore, if the dependent variable Y is coded 0, 1, or 2 and using $Y=0$ as the baseline, the probabilities of each dependent variable categories are [11]

$$P(Y=0|\boldsymbol{x}) = \frac{1}{1+e^{g_1(\boldsymbol{x})}+e^{g_2(\boldsymbol{x})}}, \quad (2)$$

$$P(Y=1|\boldsymbol{x}) = \frac{e^{g_1(\boldsymbol{x})}}{1+e^{g_1(\boldsymbol{x})}+e^{g_2(\boldsymbol{x})}}, \quad (3)$$

$$P(Y=2|\boldsymbol{x}) = \frac{e^{g_2(\boldsymbol{x})}}{1+e^{g_1(\boldsymbol{x})}+e^{g_2(\boldsymbol{x})}} \quad (4)$$

where

$$g_1(\boldsymbol{x}) = \ln\left[\frac{P(Y=1|\boldsymbol{x})}{P(Y=0|\boldsymbol{x})}\right] = \beta_{10}+\beta_{11}x_1+\beta_{12}x_2+\ldots+\beta_{1p}x_p = \boldsymbol{x}'\beta_1 \quad (5)$$

and

$$g_2(\boldsymbol{x}) = \ln\left[\frac{P(Y=2|\boldsymbol{x})}{P(Y=0|\boldsymbol{x})}\right] = \beta_{20}+\beta_{21}x_1+\beta_{22}x_2+\ldots+\beta_{2p}x_p = \boldsymbol{x}'\beta_2 \quad (6)$$

Again, the logistic regression model parameters can be determined by utilising available logistic regression software.

3. Stability probability of hard rock pillar

3.1. Hard rock pillar stability data

Hard rock pillar stability combined database utilised in this paper was mainly comprised of massive sulphide pillars with Rock Mass Ratings of between 60 and 85 while major structural features were not deemed to be an influence in pillar instability [8]. Pillar heights were measured in the direction of the major induced principal stress within the pillars and pillar widths were measured in the direction perpendicular to pillar heights. For non-square pillars, the widths were the minimum widths of the pillars. Pillar stresses were the average stresses which were calculated using tributary area theory, two-dimensional boundary element, two-dimensional finite element, and three-dimensional finite element modelling.

Furthermore, in all of the cases [2–7] analysed in the database, pillar stability are in the range from a simple “stable/failed” to a more rigorous approach based upon a five or six stage stability classification. After a review of the database and an assessment made during the process three levels of pillar stability were suggested, which were stable, unstable, and failed pillars as depicted in Table 1 which provided adequate results for the combined database.

The resulted database is given in Tables 2–4, which are basically based on Ref. [8] and further reported in Ref. [9]. The format of Tables 2–4 is based on the common practice in presenting pillar data and pillar strength in a pillar stability graph [3]. The y-axis represents a relative index of pillar loading calculated as the ratio of average induced pillar stress versus the UCS of the intact rock. Although a number of pillar strength formulae have been suggested as listed in Ref. [14], the UCS of the intact rock was used as it is an index that can be utilised in a simpler way without carrying out pillar strength estimation. The x-axis takes into account pillar shape expressed by the ratio of the pillar width to pillar height that also accounts for the increased strength provided by the shape and confinement of the pillar.

Table 2 shows that the stable pillars were constructed with width to height ratio (w/h) of 0.46–4.50. The most slender stable pillar experienced stress that was 11% of the intact rock UCS ($\sigma_p/\sigma_c=0.11$). The squattest stable pillar had a w/h ratio of 4.50 and could sustain stress up to 67% of the intact rock UCS. The w/h ratios of unstable pillars in Table 3 were 0.45–3.03. The ratio σ_p/σ_c ratio of the most slender and the squattest pillars were 16% and 58% respectively. Table 4 shows that the most slender pillar ($w/h=0.31$) could only sustain stress up to 18% of intact rock UCS and the squattest pillar ($\sigma_p/\sigma_c=2.27$) could only be loaded up to 67% of intact rock UCS.

3.2. Multinomial logistic regression model from hard rock pillar stability data

The multinomial logistic regression model from pillar stability data was developed with the independent and dependent variables listed in Table 5. With the stable case as the baseline, stability probabilities of the hard rock pillars can be calculated by using Eqs. (2)–(4) and logistic regression software and the resulted probabilities of stable, unstable, and failed can now be written as

$$P(stable) = \frac{1}{1+e^{[-5.876-5.321(w/h)+32.636(\sigma_p/\sigma_c)]}+e^{[-1.969-3.179(w/h)+18.488(\sigma_p/\sigma_c)]}}, \quad (7)$$

$$P(unstable) = \frac{e^{[-1.969-3.179(w/h)+18.488(\sigma_p/\sigma_c)]}}{1+e^{[-5.876-5.321(w/h)+32.636(\sigma_p/\sigma_c)]}+e^{[-1.969-3.179(w/h)+18.488(\sigma_p/\sigma_c)]}}, \quad (8)$$

Table 1
Description of pillar stability [8].

Pillar stability	Observed pillar condition
Stable	Minor spalling, no joint opening [4] No sign of stress induced fracturing [8]
Unstable	Showing one or more of the following signs: cracking and spalling in development and raises within the rib pillar; audible noise in the pillar; deformed drill holes; excess muck being pulled from stopes (dilution); cracking of pillars; major displacements within the pillar [3] Prominent spalling [4] Fractures also appear on the central parts of the pillar [6] Corner breaking only up to fracturing in pillar walls with fracture aperture up to 10 mm [8]
Failed	Severe spalling, pronounced opening of joints, deformation of drill holes [4] Disintegration of pillar; blocks falling out; fractures trough pillar with fracture apertures wider than 10 mm [8]