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Estimation of regional stress state and Young's modulus by back analysis of mining-induced deformation

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ABSTRACT

A method to evaluate regional stress state and Young's modulus by analyzing the mining-induced deformation of ground is proposed. The effect of changes in regional strain state due to tectonic plate motion can be accounted for using the proposed method. Mining-induced deformation can manifest itself as changes in ground surface geometry resulting from mining activity. There are three underlying factors that can contribute to the deformation, these being: displacement induced by the effect of gravity; displacement induced by the effect of horizontal regional strain; and, incremental change in the horizontal regional strain. Both regional strain and Young's modulus can be simultaneously estimated, because the displacement induced by the regional strain is independent of the Young's modulus and the displacement induced by the gravity is inversely proportional to the Young's modulus. The relative displacement arising from the mining excavation at the Torigatayama limestone mine in Japan has been measured by GPS. Both the regional stress state and Young's modulus were estimated by back analysis of the relative displacement using a 3-D finite element method. It was shown that back analysis based on changes in distance between the measurement points provides a more reliable estimation than that could be achieved based on the direct relative displacement.

1. Introduction

Determination of the state of stress in rock at both regional and local levels is the focus of much research as it is useful in earthquake prediction and in designing surface rock structures and underground caverns in mining and civil engineering projects.

The crustal stress patterns in and around the Japanese archipelago have been measured by seismological techniques, including focal mechanisms and seismic moment tensors of earthquakes [1–3]. Recently, an estimation of the 3-D crustal stress pattern over the entire Japanese archipelago has been attempted [4] by an inversion method based on the statistical inference theory [5,6]. Essentially, these methods estimate the principal direction of crustal stress on the scale of seismic source faults. From a set of estimated crustal stress patterns, global plate tectonics in and around the Japanese archipelago have been examined [7,8]. However, these seismological methods do not provide information about the magnitude of the crustal stress.

Both the stress relief technique and the hydro-fracturing technique are used in engineering projects to estimate rock stress [9,10]. Several methods using boring cores, such as the Acoustic Emissions (AE) and Deformation Rate Analysis (DRA) methods, have also been developed and widely adopted because of their simplicity and accuracy [11,12]. These methods can be used to estimate the magnitude and direction of principal stresses in rock at a specific point. Such a stress state is affected by topography and/or geological structures such as faults.

It is well known that hydraulic conductivity and the mechanical behavior of a rock mass strongly depends on the stress state. Hence with respect to the design and excavation of stable underground repositories for high-level radioactive wastes and larger mining operations, an estimation of the stress distribution in the range of several kilometers is often required. Depending on the size of these excavations, a number of stress measurements are needed to determine the stress distribution though practical considerations such as site access can limit the number of measurements that can be taken. To overcome this problem, a number of techniques have been developed that combine the results of stress measurements and numerical analyses to estimate rock stress on the scale of several kilometers [13–16]. In these methods, rock stress is a result of the superposition of gravitational stress and tectonic stress. A three-dimensional numerical stress analysis for the target district is undertaken and the tectonic stress in the target district is estimated by a back analysis based on

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the measured stress state. Next, the stress state at an arbitrary point can be estimated by forward analysis using the estimated tectonic stress. Recently, Li et al. [17] and Matsuki et al. [18] successfully estimated the local stress in a heterogeneous rock mass comprising different rock bodies intersected by a large fault. The estimation accuracy of these methods is compromised however if the stress distribution around the measurement point is disturbed due to some local heterogeneity.

The deformation behavior of rock slopes in open-pit mines is dependent on the level and direction of rock stress, especially on the ratio of horizontal stress to vertical stress, as the excavation causes stress relief and relaxation in a rock mass [15,19,20]. Conversely, the rock stress around the mine can be estimated by measuring the mining-induced deformation. It is likely that back analysis based on the mining-induced deformation would provide a more accurate estimation of rock stress on a large scale than back analysis based on stress measurements alone as it is independent of any local stress disturbances.

The continuous crustal deformation of the Japanese archipelago has been measured by a nationwide GPS network (GEONET) for over nearly a decade, and significant strain accumulation was found in inland areas and in plate boundary regions [21]. This finding suggests that the effects of changes in the tectonic strain state are also important in the estimation of the rock stress.

A method has been developed for estimating both the rock stress and Young's modulus of the ground through analyzing mining-induced deformation. The effects of changes in the tectonic strain state are accounted for in the proposed method. This paper outlines the method in three stages. First, relationships between mining-induced deformations, strain state, strain increment and elastic constants are formulated. Second, a practical method is then developed to estimate both the strain state and the elastic constant of deformation by back analysis. Third, GPS measurements at the Torigatayama limestone mine are briefly reviewed followed by an explanation of the method used to generate a 3-D FEM mesh for the limestone mine. The stress state is subsequently estimated from both the specified strain state and the specified elastic constants by back analysis. Finally, the validity of the estimations is discussed.

In this paper, traditional terms in rock engineering are used to represent the scale of the stress or strain field. Namely, the term "regional stress" refers to the stress defined in the far-field on a scale comparable to a mine, and the term "local stress" refers to the stress defined on a scale comparable to a borehole.

2. Method for estimating the regional stress state by measuring displacement

2.1. Theory of estimation

The stress state of a regional field is represented with two conditions: either the regional stress state [13,16,17,22] or regional strain state [14,18]. In this study, a uniform regional strain state technique [14] is used, and the relationship between the local stress state and given uniform regional strain will be shown.

The stress state $\boldsymbol{\sigma}(\mathbf{x})$ at a local point x can be represented by the following equation as the summation of stress states related to both gravity and regional strains.

$$\sigma(\mathbf{x}) = \sigma^0(\mathbf{x},\rho g) + \sigma^X(\mathbf{x},\varepsilon_{xx}^G) + \sigma^Y(\mathbf{x},\varepsilon_{yy}^G) + \sigma^{XY}(\mathbf{x},\gamma_{xy}^G) \tag{1}$$

where $\sigma^0(\mathbf{x},\rho g)$ is the stress state due to gravity, ρ and g are ground density and gravity acceleration, respectively, $\sigma^X(\mathbf{x},\varepsilon_{xx}^G)$, $\sigma^Y(\mathbf{x},\varepsilon_{yy}^G)$ and $\sigma^{XY}(\mathbf{x},\gamma_{xy}^G)$ are stress states arising from the given regional strains ε_{xx}^G, ε_{yy}^G and γ_{xy}^G, respectively. These are interpreted as the stress states induced by the horizontal movement of the earth's crust resulting from the tectonic plate motion. Now, suppose that there is a sufficiently large-scale region including point x and its boundary surfaces, except the ground surfaces are flat and perpendicular to each other. Then, $\sigma^0(\mathbf{x},\rho g)$ is evaluated by applying the unit weight of the ground through the region, while displacements on the side surfaces and the basal surface of the region are specified by setting them to zero. $\sigma^X(\mathbf{x},\varepsilon_{xx}^G)$, $\sigma^Y(\mathbf{x},\varepsilon_{yy}^G)$ and $\sigma^{XY}(\mathbf{x},\gamma_{xy}^G)$ are evaluated by forcing the horizontal displacement of the side surfaces as the strain state in the region is maintained equal to ε_{xx}^G, ε_{yy}^G and γ_{xy}^G. The magnitude of these regional strains is specified by back analysis of the on-site stress measurement using Eq. (1). The regional stress state can be calculated from the specified regional strain state using Hooke's law. As $\sigma^X(\mathbf{x},\varepsilon_{xx}^G)$, $\sigma^Y(\mathbf{x},\varepsilon_{yy}^G)$ and $\sigma^{XY}(\mathbf{x},\gamma_{xy}^G)$ are proportional to Young's modulus, the specified regional strains determined by back analysis are always inversely proportional to Young's modulus. Hence because of this interdependency, the values for Young's modulus and regional strain cannot be estimated. Only the product of Young's modulus and regional strain can be estimated Poisson's ratio should be varied in the estimation because all terms in Eq. (1) directly depend on Poisson's ratio.

Next, the relationship between the given regional strain and induced displacement are described. It should be noted that displacement is measured relatively and not absolutely. This means that any displacement cannot be detected if there is no change in the regional properties, including geometry, the mechanical properties of the ground, and the regional strain state. In this study, the effects of change in both the geometry and regional strain state on the elastic deformation of the ground were investigated. Changes in geometry are often caused by several factors; however, only the changes induced by mining, dumping and backfilling in an open pit mine are addressed.

Displacement indicates displacement increment over a period of time. It also indicates displacement relative to a reference point. Namely, it precisely means the relative displacement increment. However, the "relative displacement increment" is abbreviated as "relative displacement" for simplicity. Based on Eq. (1), the relative displacement at point $\mathbf{x}$ from t to $t+\Delta t$ induced by mining under a constant regional strain state can be represented by the following set of linear equations.

$$\mathbf{u}_1(\mathbf{x}) = \mathbf{u}_1^0(\mathbf{x},\rho g,E^G) + \mathbf{u}_1^X(\mathbf{x},\varepsilon_{xx}^G) + \mathbf{u}_1^Y(\mathbf{x},\varepsilon_{yy}^G) + \mathbf{u}_1^{XY}(\mathbf{x},\gamma_{xy}^G) \tag{2}$$

$$\begin{aligned}
\mathbf{u}_1^0(\mathbf{x},\rho g,E^G) &= \mathbf{u}^0(t+\Delta t,\mathbf{x},\rho g,E^G) - \mathbf{u}^0(t,\mathbf{x},\rho g,E^G)\\
\mathbf{u}_1^X(\mathbf{x},\varepsilon_{xx}^G) &= \mathbf{u}^X(t+\Delta t,\mathbf{x},\varepsilon_{xx}^G) - \mathbf{u}^X(t,\mathbf{x},\varepsilon_{xx}^G)\\
\mathbf{u}_1^Y(\mathbf{x},\varepsilon_{yy}^G) &= \mathbf{u}^Y(t+\Delta t,\mathbf{x},\varepsilon_{yy}^G) - \mathbf{u}^Y(t,\mathbf{x},\varepsilon_{yy}^G)\\
\mathbf{u}_1^{XY}(\mathbf{x},\gamma_{xy}^G) &= \mathbf{u}^{XY}(t+\Delta t,\mathbf{x},\gamma_{xy}^G) - \mathbf{u}^{XY}(t,\mathbf{x},\gamma_{xy}^G)
\end{aligned} \tag{3}$$

where $\mathbf{u}_1^0(\mathbf{x},\rho g,E^G)$ is the displacement due to gravity release by mining, $E^G$and ρg are the Young's modulus and the unit weight of the ground, respectively, and $u_1^X(\mathbf{x},\varepsilon_{xx}^G)$, $\mathbf{u}_1^Y(\mathbf{x},\varepsilon_{yy}^G)$ and $\mathbf{u}_1^{XY}(\mathbf{x},\gamma_{xy}^G)$ are the mining induced displacement under normal strains ε_{xx}^G, ε_{yy}^G and shear strain γ_{xy}^G, respectively.

In an open pit mine, both dumping and backfilling waste probably cause considerable displacement of the ground. The effects of these factors must be estimated when the amount of waste is sufficiently high or dumping and backfilling areas are very close to the displacement measurement points. It is known that tectonic plate motion continuously changes regional strain states. Strain increments on the order of 10^{-7} per year are observed by GPS measurement by the Geographical Survey Institute of Japan. The total strain increment over the course of 10 years produces changes in distances in the order of 1 mm for every 1 km. Measurement results sometimes include rigid rotation resulting from tectonic plate motion. In this study, displacements caused by plate motion as well as the effects of both dumping and backfilling are also considered. From t to $t+\Delta t$, relative displacements $\mathbf{u}_2(\mathbf{x})$