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A transverse isotropic constitutive model for the aortic valve tissue incorporating rate-dependency and fibre dispersion: Application to biaxial deformation

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ABSTRACT

This paper presents a continuum-based transverse isotropic model incorporating rate-dependency and fibre dispersion, applied to the planar biaxial deformation of aortic valve (AV) specimens under various stretch rates. The rate dependency of the mechanical behaviour of the AV tissue under biaxial deformation, the (pseudo-) invariants of the right Cauchy-Green deformation-rate tensor $\dot{\mathbf{C}}$ associated with fibre dispersion, and a new fibre orientation density function motivated by fibre kinematics are presented for the first time. It is shown that the model captures the experimentally observed deformation of the specimens, and characterises a shear-thinning behaviour associated with the dissipative (viscous) kinematics of the matrix and the fibres. The application of the model for predicting the deformation behaviour of the AV under physiological rates is illustrated and an example of the predicted $\sigma - \lambda$ curves is presented. While the development of the model was principally motivated by the AV biomechanics requisites, the comprehensive theoretical approach employed in the study renders the model suitable for application to other fibrous soft tissues that possess similar rate-dependent and structural attributes.

1. Introduction

The structural composition of the aortic valve (AV) may be viewed as arrangements of elastin and collagen fibres embedded within the glycosaminoglycans (GAGs) ground matrix – see for example Anssari-Benam et al. (2011a) and Anssari-Benam et al. (2016). This structure endows the AV tissue with marked directional- and rate-dependency in its mechanical properties, i.e. anisotropy and viscoelasticity (Anssari-Benam et al., 2017). In our previous study we devised a new continuum-based model to capture these attributes by considering the mechanical contribution of the '*isotropic matrix*', the circumferentially aligned collagen fibres and the viscous effects of GAGs (Anssari-Benam et al., 2017). The viscous term was introduced as an explicit function of the stretch rate, while the elastic contribution was accounted for using a *Holzapfel-type* additive split of the elastic energy functions pertaining to the matrix and the fibres, respectively. In order to characterise the contribution of the '*isotropic matrix*' more accurately, we recently proposed a structurally motivated energy function for the mechanical contribution of the elastin network to the overall load-bearing capacity of the AV and its non-linear mechanical behaviour attributes under tensile deformation (Anssari-Benam and Bucchi, 2017). However, as with other collagenous soft tissues, collagen fibres are the principal structural elements that confer anisotropy and augmented mechanical strength to the AV matrix. Therefore, continuum-based models of the mechanical behaviour of the AV should properly incorporate how the collagen fibres are embedded within the tissue structure and correctly account for the ensuing material symmetry.

Macroscopic (see, e.g., Rock et al., 2014) and microscopic (see, e.g., Billiar and Sacks, 1997; Sacks et al., 1998) studies of the AV structure have well established that collagen fibres are principally aligned along the circumferential direction in relation to each AV leaflet. The circumferential and radial loading directions are defined in Fig. 1. Therefore, a suitable class of anisotropy to model the mechanical behaviour of the AV may be considered as '*transverse isotropy*' (Freed et al., 2005; Anssari-Benam et al., 2017). From a continuum mechanics point of view, this structural attribute of the AV tissue is rather convenient, because in-plane uniaxial and biaxial tensile tests do provide the required datasets that facilitate validation of models of transverse

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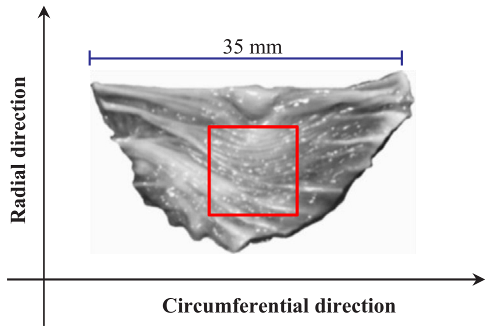

Fig. 1. Typical intact porcine AV leaflet used in this study. The principal loading directions of the leaflet, i.e. the circumferential and radial loading directions, are shown in reference to the leaflet. Square specimens (12 mm × 12 mm) were prepared from the central region of the AV leaflet for in-plane biaxial tensile tests.

isotropy. Those tests, however, do not provide enough independent datasets that are required to characterise models with higher class of anisotropy, e.g. two preferred directions of fibre families, as a function of invariants I_1, I_4 and I_6 (see Holzapfel and Ogden, 2009 and Ogden, 2009). However, the circumferential alignment of collagen fibres in AV tissue is not a perfect alignment, with a degree of fibre dispersion around the circumferential direction (Billiar and Sacks, 1997; Sacks et al., 1998). It is therefore important to incorporate this structural feature into the continuum-based models of the AV, for a more accurate characterisation of the biomechanical behaviour of the tissue.

The pioneering work of Freed et al. (2005) introduced the incorporation of fibre dispersion into the formulation of a continuum-based transverse isotropic model of the AV by devising a material tensor. This work was preceded by that of Sacks (2003), where the angular distribution of the collagen fibres within the AV tissue was accounted for via an '*angular integration*' directly incorporating a probability density function into the second Piola-Kirchhoff stress tensor. However, those models did not include the ('*viscoelastic*') rate effects. To our knowledge, Anssari-Benam et al. (2017) presented the first continuum-based rate-dependent transverse isotropic model for application to the AV. That work, however, assumed perfect alignment of the fibres along the circumferential direction and did not account for fibre dispersion.

In this paper, we extend the model presented by Anssari-Benam et al. (2017) to incorporate fibre dispersion into rate-dependent continuum-based modelling of the AV. In doing so, we derive and introduce a new fibre orientation density function which is motivated based on the kinematics of fibres in tissue deformation. We show how the introduced measure of fibre dispersion into the model formulation may be treated as a *phenomenological* concept that is characterised by fitting the stress-stretch data to the model, without having direct structural data on the distribution of fibres within the specimens. We also define and present the invariants of the right Cauchy-Green deformation-rate tensor $\dot{\mathbf{C}}$ associated with fibre dispersion. The model is then applied to the experimental data obtained from porcine AV specimens under planar biaxial tensile tests at two different displacement ratios and four stretch rates covering a range of 1000-fold. The rate-dependency of the tensile biaxial deformation of the AV specimens is presented, and it will be shown that the model successfully captures and characterises this behaviour.

2. Continuum mechanics framework

The framework within which we devise the constitutive relationship between stress and strain tensors is similar to that of our previous work (Anssari-Benam et al., 2017). Accordingly, for an incompressible rate-dependent continuum, the second Piola-Kirchhoff stress tensor $\mathbf{S}$ may be expressed as (Pioletti et al., 1998; Limbert and Middleton, 2004; Vogel et al., 2017) [1]:

$$\mathbf{S}(\mathbf{C}, \dot{\mathbf{C}}) = 2\frac{\partial W_e}{\partial \mathbf{C}} + 2\frac{\partial W_v}{\partial \dot{\mathbf{C}}} - p\mathbf{C}^{-1}, \tag{1}$$

where $\mathbf{C}$ is the right Cauchy-Green tensor and $\dot{\mathbf{C}}$ is its time derivative, W_e and W_v are two distinct thermodynamics potentials referred to as the elastic strain energy and the (viscous) dissipation functions, respectively, and p is an arbitrary Lagrange multiplier enforcing incompressibility. Note that W_e and W_v may be described as functions of ($\mathbf{C}$) and ($\mathbf{C}$, $\dot{\mathbf{C}}$), respectively (Limbert and Middleton, 2004), i.e.:

$$\begin{cases} W_e = W_e(\mathbf{C}), \\ W_v = W_v(\mathbf{C}, \dot{\mathbf{C}}), \end{cases} \tag{2}$$

In the case of transverse isotropy, W_e and W_v will be functions of $W_e(\mathbf{C}, \mathbf{M})$ and $W_v(\mathbf{C}, \dot{\mathbf{C}}, \mathbf{M})$, where $\mathbf{M}$ denotes the preferred mean orientation in the reference configuration. It follows that W_e and W_v may be expressed as a function of five and seventeen invariants, respectively:

$$\begin{cases} W_e = W_e(I_1, \dots, I_5), \\ W_v = W_v(I_1, \dots, I_5, J_1, \dots, J_{12}), \end{cases} \tag{3}$$

where I_i, $i = 1,\dots,5$, and J_j, $j = 1,\dots, 12$ are the respective invariants; the mathematical definition of which is given in Appendix A.

2.1. Incorporation of fibre orientation dispersion

In the presence of distributed fibres within the continuum, following the frameworks presented by Gasser et al. (2006) and Holzapfel and Ogden (2010), it is assumed that W_e and W_v are not only functions of $\mathbf{C}$, $\dot{\mathbf{C}}$ and $\mathbf{M}$, but are indeed also a function of the *structure tensor* $\mathbf{H}$, which accounts for the distribution of fibre orientation around a preferred mean direction. Accordingly, the existence of a fibre orientation density function, say $R(\mathbf{M})$, is postulated such that it characterises the distribution of fibres with respect to $\mathbf{M}$, where $\mathbf{M}$ is a unit vector representing the mean general preferred direction of the fibre family in the Cartesian coordinate system, as depicted in Fig. 2. It is given by:

$$\mathbf{M}(\theta, \varphi) = \cos\theta\cos\varphi\mathbf{e}_1 + \cos\theta\sin\varphi\mathbf{e}_2 + \sin\theta\mathbf{e}_3, \tag{4}$$

where $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ denote the unit vectors of the coordinate system, and the angles θ and φ are defined in Fig. 2 (note that $-\pi/2 \le \theta \le \pi/2$ and $0 \le \varphi \le 2\pi$).

The fibre orientation density function $R(\mathbf{M})$ is defined such that the fraction of fibres oriented within $[(\theta, \theta + d\theta), (\varphi, \varphi + d\varphi)]$ is characterised by $R(\mathbf{M})\cos\theta d\theta d\varphi$, while satisfying the condition of symmetry $R(\mathbf{M}) = R(-\mathbf{M})$. In addition, $R(\mathbf{M})$ is normalised according to:

$$\frac{1}{4\pi}\int_{\Omega} R(\mathbf{M})d\omega = 1, \tag{5}$$

where $\Omega = \{\mathbf{M} \in \Re^3, |\mathbf{M}| = 1\}$ is a unit sphere and $d\omega$ is the surface area element given by $d\omega = \cos\theta d\theta d\varphi$. Thus:

$$\frac{1}{4\pi}\int_{\varphi=0}^{2\pi}\int_{\theta=-\pi/2}^{\pi/2} R(\mathbf{M})\cos\theta d\theta d\varphi = 1. \tag{6}$$

Let $\mathbf{N}$ represent the direction of an arbitrary fibre from the fibre family, a unit vector given by:

$$\mathbf{N}(\Theta, \Phi) = \cos\Theta\cos\Phi\mathbf{e}_1 + \cos\Theta\sin\Phi\mathbf{e}_2 + \sin\Theta\mathbf{e}_3, \tag{7}$$

as shown in Fig. 2. Note that the angles Θ and Φ are defined in a similar way as θ and φ (Fig. 2). The structure tensor $\mathbf{H}$ for the fibre family whose mean direction and position are represented by $\mathbf{M}$ and $\mathbf{N}$, respectively, is thus defined as:

[1] The theoretical underpinning of 'rate-type' viscoelastic models, whereby the viscoelastic response is determined by a stored energy function and a rate of dissipation function, has also been extensively examined in a different context through the works of Rajagopal and co-workers (see, e.g., Rajagopal and Srinivasa, 2000).