ADAPTIVE CONTROL OF A CONTINUOUS STIRRED TANK REACTOR BY TWO FEEDBACK CONTROLLERS

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Abstract: The paper deals with adaptive control of a continuous stirred tank reactor (CSTR). A nonlinear model of the process is approximated by a continuous-time external linear model. The parameters of the CT external linear model of the process are estimated using a corresponding delta model. The control system with two feedback controllers is considered. The controller design is based on the polynomial approach. The resulting proper controllers ensure stability of the control system as well as asymptotic tracking of step references and step load disturbance attenuation. The adaptive control is tested on the nonlinear model of the CSTR with a consecutive exothermic reaction.

Keywords: Adaptive control, continuous-time model, delta model, parameter estimation, polynomial method.

1. INTRODUCTION

Continuous stirred tank reactors (CSTRs) belong to a class of nonlinear systems where both steady-state and dynamic behaviour are nonlinear. Their models are derived and described in e.g. (Ogunnaike and Ray, 1994), (Schmidt, 2005) and (Corriou, 2004). The process nonlinearities may cause difficulties when controlling using conventional controllers with fixed parameters. One possible method to cope with this problem is using adaptive strategies based on an appropriate choice of an external linear model (ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating of controller‘s parameters.

The control itself can be either continuous-time or discrete. While for design of a continuous-time controller, it is necessary to know a continuous-time ELM and its parameters, a discrete-time controller requires knowledge of a discrete ELM. Experiences of authors in the field of control of nonlinear technological processes indicate that the continuous-time (CT)approach gives better results when controlling processes with strong nonlinearities. In the case of discrete control in order to cope with the nonlinearity, it is necessary to sample signals very frequently. However, it is well known from the properties of transfer functions in the z-domain that a sampling period cannot be shortened too much.

For the CT ELM parameters estimation, either the direct method or application of an external delta model with the same structure as the CT model can be used. The procedure based on direct CT ELM parameter estimation was described in (Dostál *et al.*, 2001).

The basics of delta models have been described in e.g. (Middleton and Goodwin, 1990), (Mukhopadhyay *et al.*, 1992) and (Goodwin *et al.*, 2001). Although delta models belong into discrete models, they do not have such disadvantageous properties connected with shortening of a sampling period as discrete z-models. In addition, parameters of delta models can directly be estimated from sampled signals. Moreover, it can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process). Complete description and experimental verification can be found in (Stericker and Sinha, 1993).

This contribution deals with continuous-time adaptive control of the CSTR as a non-linear single input – single output process. The parameters of its CT ELM are obtained via corresponding delta model parameter estimation. The control system with two feedback controllers is used according to (Ortega and Kelly, 1984). This set-up gives better control results for the reference tracking than using only a feedback controller. Input signals for the control system are step references and step disturbances injected into the input of the controlled process. The resulting controllers derived using polynomial method (Kučera, 1993) guarantee stability of the control system, asymptotic tracking of step references and step load disturbances attenuation. The approach is tested on a nonlinear model of the CSTR with a consecutive exothermic reaction.

2. CT EXTERNAL LINEAR MODEL

The CT external linear model (ELM) is chosen on the basis of some preliminary knowledge of dynamic behaviour of the controlled nonlinear process. This model is described in the time domain by differential equation

$$a(\sigma)\, y(t) = b(\sigma)\, u(t) \tag{1}$$

where $\sigma = d/dt$ is the derivative operator and a, b are polynomials in σ. Considering zero initial conditions, and, using the Laplace transform, the ELM is represented in the complex domain by the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b(s)}{a(s)} \tag{2}$$

where s is the complex variable and both a and b are coprime polynomials in s. The transfer function (2) is considered to be proper ($\deg b \le \deg a$).

3. DELTA MODEL

Establish the delta operator defined by

$$\delta = \frac{q-1}{T_0} \tag{3}$$

where q is the forward shift operator and T_0 is the sampling period. When the sampling period is shortened, then, the delta operator approaches the derivative operator σ so that

$$\lim_{T_0 \to 0} \delta = \sigma \tag{4}$$

and, the δ-model

$$a'(\delta)\, y(t') = b'(\delta)\, u(t') \tag{5}$$

approaches the continuous-time model (1) as shown in (Stericker and Sinha, 1993).

Here, t' is the discrete time, and, a', b' are polynomials in δ.

4. DELTA MODEL PARAMETER ESTIMATION

Substituting $t' = k - n$ where $k \ge n$, equation (5) may be rewriten as

$$\begin{aligned} \delta^n y(k-n) &= b'_m \delta^m u(k-n) + \dots + b'_1 \delta u(k-n) + \\ &+ b'_0 u(k-n) - a'_{n-1} \delta^{n-1} y(k-n) - \dots \\ \dots &- a'_{n-1} \delta^{n-1} y(k-n) - \dots - a'_1 \delta y(k-n) - a'_0 y(k-n) \end{aligned} \tag{6}$$

The terms in (6) can be expressed as

$$\delta^i y(k-n) = \sum_{j=0}^{i} \frac{(-1)^j}{T_0^i} \binom{i}{j} y(k-n+i-j) \tag{7}$$

for $i = 0, 1, \dots, n$, and,

$$\delta^l u(k-n) = \sum_{j=0}^{l} \frac{(-1)^j}{T_0^l} \binom{l}{j} u(k-n+l-j) \tag{8}$$

for $l = 0, 1, \dots, m$.

Obviously, an actual value of the controlled output $y(k)$ is included only in the term on the left side of (6) (for $i = n$ in (7)). Now, denoting

$$\begin{aligned} \varphi_y(k) &= \delta^n y(k-n) \\ \varphi_y(k-1) &= \delta^{n-1} y(k-n), \dots \\ \dots, \varphi_y(k-n+1) &= \delta y(k-n), \\ \varphi_y(k-n) &= y(k-n) \\ \varphi_u(k-n+m) &= \delta^m u(k-n), \dots \\ \dots, \varphi_u(k-n+1) &= \delta u(k-n), \\ \varphi_u(k-n) &= u(k-n) \end{aligned} \tag{9}$$

and, introducing the regression vector

$$\begin{aligned} \boldsymbol{\Phi}_\delta^T(k-1) = \big[&-\varphi_y(k-n) - \varphi_y(k-n+1) \dots - \varphi_y(k-1) \\ &\varphi_u(k-n)\ \varphi_u(k-n+1) \dots \varphi_u(k-n+m) \big] \end{aligned} \tag{10}$$

then, the parameter vector

$$\boldsymbol{\Theta}_\delta^T = \left[a'_0\ a'_1 \dots a'_{n-1}\ b'_0\ b'_1 \dots b'_m \right] \tag{11}$$

can be estimated recursively from the regression (ARX) model

$$\varphi_y(k) = \boldsymbol{\Theta}_\delta^T(k) \boldsymbol{\Phi}_\delta(k-1) + \varepsilon(k) \tag{12}$$

where $\varepsilon(k)$ is the non-measurable random component. For a small sampling interval T_0, the estimated parameters reach the parameters of the CT model so that

$$\begin{aligned} b'_j &\to b_j, \quad j = 0, 1, \dots m \\ a'_i &\to a_i, \quad i = 0, 1, \dots n-1 \end{aligned} \tag{13}$$

5. CONTROL SYSTEM DESCRIPTION

The control system with two feedback controllers is depicted in Fig. 1. In the scheme, w is the reference signal, v denotes the load disturbance, e is the tracking error, u_0 is the output of the controller, y is the controlled output and u is the control input.