Numerical study of temperature effects on the poro-viscoelastic behavior of articular cartilage

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ABSTRACT

This paper presents a new approach to study the effects of temperature on the poro- elastic and viscoelastic behavior of articular cartilage. Biphasic solid-fluid mixture theory is applied to study the poro-mechanical behavior of articular cartilage in a fully saturated state. The balance of linear momentum, mass, and energy are considered to describe deformation of the solid skeleton, pore fluid pressure, and temperature distribution in the mixture. The mechanical model assumes both linear elastic and viscoelastic isotropic materials, infinitesimal strain theory, and a time-dependent response. The influence of temperature on the mixture behavior is modeled through temperature dependent mass density and volumetric thermal strain. The fluid flow through the porous medium is described by the Darcy's law. The stress-strain relation for time-dependent viscoelastic deformation in the solid skeleton is described using the generalized Maxwell model. A verification example is presented to illustrate accuracy and efficiency of the developed finite element model. The influence of temperature is studied through examining the behavior of articular cartilage for confined and unconfined boundary conditions. Furthermore, articular cartilage under partial loading condition is modeled to investigate the deformation, pore fluid pressure, and temperature dissipation processes. The results suggest significant impacts of temperature on both poro- elastic and viscoelastic behavior of articular cartilage.

1. Introduction

Diarthodial joints are the most common movable skeletal joints which are specifically characterized by some common structural features including: a layer of fibrocartilage that covers the opposing bony surfaces; a lubricating synovial fluid within the joint cavity; and an enclosing fibrous capsule which is lined with an active tissue (i.e. the synovium) (Mow et al., 1993). Fig. 1 illustrates the structure of knee joint as an example of diarthodial joints.

Articular cartilage is a thin connective fibrocartilage (Fig. 1) that primarily consists of water, collagens, and proteoglycans. With 80% of the wet weight, water is the dominant constituent of articular cartilage. The network of collagen, which constitutes 60% of the dry weight, together with water increase the strength of the tissue to transmit high instantaneous loads. For smooth joint articulation, articular cartilage provides a lubricated surface that facilitates load transmission to the underlying subchondral bone (Sophia Fox et al., 2009).

The mechanical behavior of articular cartilage has been studied using different models. Pena et al. (2005) studied the effect of meniscal tears and meniscectomies on knee joints considering cartilage as a linear elastic isotropic and homogeneous material. Viscoelastic models

are widely used to study the influences of an interstitial fluid on soft biological tissues (Mak, 1986; Shirazi et al., 2008; Gupta et al., 2009; Gu and Li, 2011). Poro-elastic and poro-viscoelastic models are developed to account for fluid flow in articular cartilage where biphasic mixture theory is used to describe deformation of the solid and the solid-fluid interactions (Huang et al., 2003; Netti et al., 2003; Hoang and Abousleiman, 2009; Tomic et al., 2014).

While most current models have studied the poro- elastic and viscoelastic behavior of articular cartilage, experimental studies (June and Fyhrie, 2010) revealed that increasing temperature affects the behavior of both fluid and solid phases. Increasing temperature increases equilibrium stiffness and stress relaxation at higher temperatures, and decreases fluid viscosity. A comprehensive understanding of articular cartilage mechanics requires considering the effects of temperature. This study aims to investigate the influence of temperature on the poro-elastic and viscoelastic behavior of articular cartilage.

In this paper, a finite element model is developed to explore the thermal effects coupled with the poro- elastic and viscoelastic behavior of articular cartilage. The poro-mechanical behavior of a fully saturated articular cartilage is modeled through the solid-fluid mixture theory. The mechanical deformation of the solid skeleton, fluid flow, and
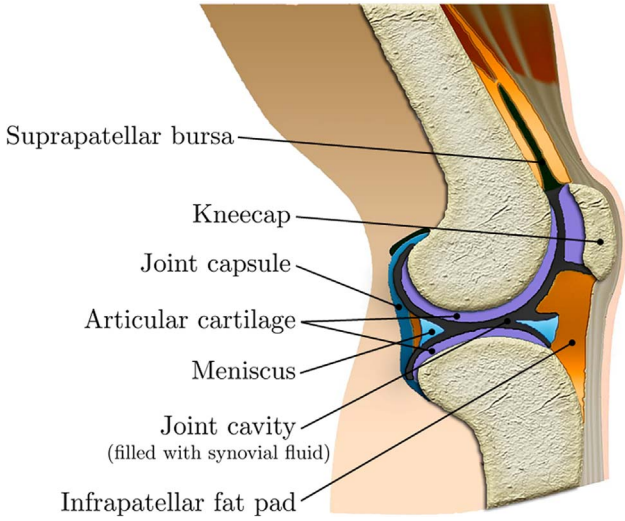

Fig. 1. Schematic diagram of the knee joint.

temperature distribution in the solid-fluid mixture are modeled by the balances of linear momentum, mass, and energy. The mechanical behavior of the solid skeleton is described by both linear elastic and viscoelastic isotropic materials, infinitesimal strain theory, and a transient response. The motion of fluid through the porous medium is described by the Darcy's law. The influence of temperature on the mixture behavior is modeled by temperature dependent mass density and volumetric thermal strain. The governing equations are discretized in time with an implicit backward Euler scheme and in space by a standard Galerkin finite element method. The accuracy and efficiency of the finite element model are evaluated using analytical reference solutions.

The remainder of the paper is organized as follows: in Section 2, the methodology for theoretical formulation of the thermo-poro- elastic and viscoelastic models is elaborated. The numerical implementation and finite element analysis are described in Section 3. In Section 4, the effects of material type, temperature, and boundary conditions on the behavior of articular cartilage are discussed. Insights gained from the results and areas for future research are summarized in Section 5.

2. Methodology

The proposed thermo-poro-mechanical model predicts the coupled thermo-poro- elastic and viscoelastic behavior of articular cartilage. The model, developed based on the continuum theory of mixtures, describes the mechanical deformation, pore fluid pressure and thermal phenomena in a fully saturated articular cartilage. For more information about the basic theory of mixtures and continuum mechanics of porous media the reader is referred to Boer (2006). The displacement of solid skeleton is modeled by the balance of linear momentum. The flow of fluid though the porous medium is described by the Darcy's law. The temperature evolution in the solid-fluid mixture is modeled by the balance of energy. The model for the thermo-poro- elastic and viscoelastic behavior of the saturated articular cartilage is described below.

2.1. Basic definition of mixture

Consider a porous medium consisting of a porous solid phase, denoted by s, with pore spaces occupied by a fluid, denoted by f. The medium, therefore, is characterized by two phases. At time t, the current position of an α phase is defined by $\mathbf{x}^\alpha$ where α represents either the solid or the fluid phase. Throughout the paper, any variable appearing in the form of $(\bullet)^\alpha$ represents the $(\bullet)$ property of the α phase. The volumetric fraction occupied by the α phase is defined by:

$$n^\alpha(\mathbf{x}^\alpha, t) = \frac{dv^\alpha}{dv}, \quad dv = J^\alpha dV^\alpha, \quad J^\alpha = \det \mathbf{F}^\alpha, \tag{1}$$

where dV^α is the initial differential volume, dv^α is the current differential volume, dv is the current total differential volume of the mixture, J^α is the Jacobian of the deformation matrix, and $\mathbf{F}^\alpha$ is the deformation gradient (Boer, 2006). In biphasic mixture theory, $n^s + n^f = 1$ where n^s and n^f are volume fractions of the solid and fluid phases, respectively. The real ($\rho^{\alpha R}$) and partial (ρ^α) mass densities of the α phase and the total mass density, ρ, are defined as follows:

$$
\begin{aligned}
\rho^{\alpha R}(\mathbf{x}^\alpha, t) &= \frac{dm^\alpha}{dv^\alpha}, \\
\rho^\alpha(\mathbf{x}^\alpha, t) &= \frac{dm^\alpha}{dv} = \rho^{\alpha R}(\mathbf{x}^\alpha, t)\, n^\alpha(\mathbf{x}^\alpha, t), \\
\rho(\mathbf{x}^\alpha, t) &= \rho^s(\mathbf{x}^\alpha, t) + \rho^f(\mathbf{x}^\alpha, t),
\end{aligned}
\tag{2}
$$

where dm^α is the differential mass.

The material time derivative of any spatial field, $\psi^\alpha(\mathbf{x}^\alpha, t)$, for the α phase can be split into the local and convected terms as follows (Boer, 2006):

$$
\begin{aligned}
\frac{D^\alpha \psi^\alpha(\mathbf{x}^\alpha, t)}{Dt} &= \frac{\partial \psi^\alpha(\mathbf{x}^\alpha, t)}{\partial t} + \frac{\partial \psi^\alpha(\mathbf{x}^\alpha, t)}{\partial \mathbf{x}^\alpha} \frac{\partial \mathbf{x}^\alpha}{\partial t} \\
&= \frac{\partial \psi^\alpha}{\partial t} + \nabla \psi^\alpha \cdot \mathbf{v}^\alpha,
\end{aligned}
\tag{3}
$$

where $\mathbf{v}^\alpha$ is the velocity vector.

2.2. Balance of mass

For the α phase, the balance of mass is defined as follows:

$$\frac{D^\alpha \rho^\alpha}{Dt} + \rho^\alpha \nabla \cdot \mathbf{v}^\alpha = \gamma^\alpha, \tag{4}$$

where $\frac{D^\alpha \rho^\alpha}{Dt}$ is the material time derivative of ρ^α, γ^α is the mass supply, and $\nabla \cdot (\bullet) = \frac{\partial (\bullet)_i}{\partial x_i}$ is the divergence of a field. Considering volumetric thermal expansion for the α phase, the temperature dependent real mass density is expressed as follows (Lewis and Schrefler, 1999):

$$\frac{1}{\rho^{\alpha R}} \frac{D^\alpha \rho^{\alpha R}}{Dt} = \frac{1}{K^\alpha} \frac{D^\alpha p^\alpha}{Dt} - 3\overline{\alpha}_\theta^\alpha \frac{D^\alpha \theta^\alpha}{Dt}, \tag{5}$$

where K^α is the bulk modulus, p^α is the pressure, $\overline{\alpha}_\theta^\alpha$ is the thermal expansion coefficient, and θ^α is the temperature field. From material time derivative of the fluid motion with respect to the motion of the solid phase, the following equations are derived (Boer, 2006):

$$
\begin{aligned}
\frac{D^f n^f}{Dt} &= \frac{D^s n^f}{Dt} + \nabla n^f \cdot \tilde{\mathbf{v}}^f, \\
\frac{D^f p^f}{Dt} &= \frac{D^s p^f}{Dt} + \nabla p^f \cdot \tilde{\mathbf{v}}^f, \\
\frac{D^f \theta^f}{Dt} &= \frac{D^s \theta^f}{Dt} + \nabla \theta^f \cdot \tilde{\mathbf{v}}^f,
\end{aligned}
\tag{6}
$$

where $\tilde{\mathbf{v}}^f = \mathbf{v}^f - \mathbf{v}^s$ is the relative velocity of the fluid phase with respect to the motion of the solid phase. Use of Eqs. (2), (5) and (6) in (4), the balance of mass for the mixture, with respect to the solid phase motion, which accounts for the volumetric thermal expansion in both solid and fluid phases, is expressed as follows:

$$
\begin{aligned}
&\frac{n^s}{K^s} \frac{D^s p^s}{Dt} + \frac{n^f}{K^f} \frac{D^s p^f}{Dt} + \frac{1}{K^f} \nabla p^f \cdot (n^f \tilde{\mathbf{v}}^f) \\
&\quad - 3n^s \overline{\alpha}_\theta^s \frac{D^s \theta^s}{Dt} - 3n^f \overline{\alpha}_\theta^f \left(\frac{D^s \theta^f}{Dt} + \nabla \theta^f \cdot \tilde{\mathbf{v}}^f \right) \\
&\quad + \nabla \cdot \mathbf{v}^s + \nabla \cdot (n^f \tilde{\mathbf{v}}^f) = \frac{\gamma^s}{\rho^{sR}} + \frac{\gamma^f}{\rho^{fR}}.
\end{aligned}
\tag{7}
$$