SYNTHESIS OF ROBUST DISCRETE-TIME SYSTEMS BASED ON COMPARISON WITH STOCHASTIC MODEL [1]

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Abstract: The paper considers a class of linear discrete-time systems with uncertain parameters. New approach to synthesis of robust stabilizing control is proposed. This approach consists of two steps. First the stochastic comparison system with multiplicative noises is constructed such that if this stochastic system is stable in the mean square then the original system with uncertain parameters is robustly stable. Second the stabilizing control problem for the comparison system is solved. To find the gain matrix of the stabilizing controller in the case of state feedback the LMI based algorithm is given and in the case of static output feedback new method and convergent iteration algorithm are obtained.

Keywords: Discrete-time system, uncertain parameters, stochastic system, comparison system, robust control, stabilizing control, state feedback, output feedback.

1. INTRODUCTION

The study of the systems with uncertain parameters is one from the main directions of the modern control theory called robust stability and control theory. In this theory there exist several approaches to describe the uncertainty models (Boyd *et al.*, 1994; Polyak and Shcherbakov, 2002). For the class of linear systems the affine and polytopic models have wide spreading. On the one hand these models allow effectively use the semidefinite programming technique in particular the linear matrix inequalities (LMI) technique (Boyd *et al.*, 1994; Polyak and Shcherbakov, 2002; Balandin and Kogan, 2007). On the other hand if the uncertainty vector is p - dimensional then this approach requires to solve 2^p m - dimensional linear matrix inequalities. It is clear that because such high dimension these models cannot be attractive especially in control engineering practice.

Intensive flow of researches based on semidefinite programming ideas has been reduced attention to other possible approaches (Bernstein, 1987; Barmish and Lagoa, 1997; Kan, 2000; Polyak and Shcherbakov, 2005). Bernstein (1987) proposed an interesting idea based on construction of stochastic Ito diffusion process, such that robust stability of a system with uncertain parameters follows from its stochastic stability. In this case the dimension of the problem is not dependent on the number of uncertain parameters. Unfortunately this idea was forgotten because it leads to nonstandard matrix quadratic equation for which the methods of solution were obtained only much years later (AitRami and ElGhaoui, 1996; AitRami and Zhou, 2000).

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These methods based on LMI optimization approach applicable if full state vector is available for controller but the complexity of the problem does not decrease if only observable output vector available for controller. The researches of the last years (Syrmos *et al.*, 1997; Garcia *et al.*, 2003; Polyak and Shcherbakov, 2005) show that this problem is not convex and its solution in principle can be not obtained by a simple way. Several attempts of some convex approximations usually lead to algorithms which do not guarantee convergence (Pakshin, 1997; Pakshin and Retinsky, 2005; Polyak and Shcherbakov, 2005). These facts attract attention to development of convergent iteration algorithms (Yu, 2004).

In this paper the new approach to synthesis of robust stabilizing control for linear discrete-time systems with uncertain parameters is proposed. This approach consists of two steps. First we construct the stochastic comparison system with the following property: if this system is stable in the mean square then considered system with uncertain parameters is robustly stable. Second the stabilizing control problem for the comparison system is solved. To find the gain matrix of the stabilizing controller in the case of state feedback an LMI based algorithm is given and in the case of static output feedback the new method and convergent iteration algorithm are obtained. All the proofs of theorems below are omitted because limited space.

2. STATEMENT OF THE PROBLEM

Consider the linear discrete-time uncertain system described by the following equations:

$$x_{n+1} = Ax_n + Bu_n + \sum_{i=0}^{p} \sigma_i(n)(A_i x_n + B_i u_n),\ y_n = Cx_n, \qquad (1)$$

where x_n is m - dimensional state vector; u_n is k - dimensional control vector; y_n is r - dimensional output vector; $A, A_i \quad (i = 1, \dots, p)$ are $m \times m$ matrices; $B, B_i \quad (i = 1, \dots, p)$ is $m \times k$ matrices; C is $r \times m$ matrix; $\sigma_i(n)$ are variables which describe the uncertainties of a parameters, it is known only that these variables are bounded:

$$|\sigma_i(n)| \le \delta_i \qquad i = 1, \dots, p. \qquad (2)$$

Consider the following problems:

- find the state feedback control

$$u_n = -Kx_n, \qquad (3)$$

  providing exponential stability of closed loop system (1) under parameters uncertainties satisfying inequalities (2) (robust state feedback stabilization);

- find the output feedback control

$$u_n = -Fy_n, \qquad (4)$$

  providing exponential stability of closed loop system (1) under parameters uncertainties satisfying inequalities (2) (robust output feedback stabilization). Here K and F are $k \times m$ and $k \times r$ gain matrices.

3. STOCHASTIC COMPARISON MODEL

Together with (1) consider the stochastic discrete-time system

$$x_{n+1} = A_{c\alpha} x_n + \sum_{i=1}^{p} \gamma_i A_{ci} x_n v_i(n),\ y_n = Cx_n, (5)$$

where $A_{c\alpha} = (1+\alpha)^{1/2} A_c,\ A_c = (A - BG), \alpha > 0,\ A_{ci} = A_i - B_i G$; $v_i(n)$ are components of p - dimensional Gaussian white noise $v(n)$ with identity covariance matrix, γ_i are positive scalars, $i = 1, \dots, p$. Take the standard assumption that the noise $v(n)$ does not depend on the initial state of the system (5).

The following statements establish connection between stability of the system (5) and robust stabilization of the system (1). Denote $\mathbb{S}^m$ the space of real valued symmetric matrices.

Theorem 1. Let for some $\alpha > 0,\ \gamma > 0$ there exists positive definite solution $P \in S^m$ of the matrix equation

$$A_{c\alpha}^T P A_{c\alpha} - P + \sum_{i=1}^{p} \gamma_i^2 A_{ci}^T P A_{ci} + \gamma I = 0, \qquad (6)$$

satisfying condition

$$(\alpha - \sum_{i=1}^{p} \frac{\delta_i^2}{\Gamma_i}) A_c^T P A_c + \gamma I > 0,$$
$$0 < \Gamma_i \le \gamma_i^2 - \delta_i (\sum_{j \ne i}^{p} \delta_j + \delta_i) \quad i = 1, \dots, p. \ (7)$$

Then the control law

$$u_n = -Gx_n, \qquad (8)$$

provides robust stabilization of the system (1).

Corollary 1. (Stochastic comparison model). Consider the stochastic system

$$x_{n+1} = A_\alpha x_n + B_\alpha u_n + \sum_{i=1}^{p} \gamma_i (A_i x_n + B_i u_n) v_i(n), \quad y_n = Cx_n, \qquad (9)$$

where $A_\alpha = (1+\alpha)^{1/2} A,\ \ B_\alpha = (1+\alpha)^{1/2} B$. Let