SELF-TUNING CONTROL BASED ON GENERALIZED MINIMUM VARIANCE CRITERION.

Anna Patete ,[1] **Katsuhisa Furuta** ,[1] **Masayoshi Tomizuka**

Department of Advanced Multidisciplinary Engineering.
E-mail: apatete@furutalab.k.dendai.ac.jp
Department of Computer and Systems Engineering.
E-mail: furuta@k.dendai.ac.jp
Tokyo Denki University, Hatoyama-cho, Hiki-gun,
Saitama, 350-0394, Japan. Fax: +81-49-296-3286.
Department of Mechanical Engineering.
University of California, Berkeley, CA 94720, USA.
Fax:+1-510-643-5599. E-mail: tomizuka@me.berkeley.edu

Abstract: The stability of adaptive control systems has been studied extensively for minimum phase systems, mainly for model reference adaptive systems, but complete stability proof for non-minimum phase systems have not been given. In this paper, the stability of two types of self-tuning controllers for discrete time minimum and non-minimum phase plants is studied, namely: recursive estimation of the implicit self-tuning controller parameters based on generalized minimum variance criterion (REGMVC), and another based on generalized minimum variance criterion - β equivalent control approach (REGMVC-β). Stability of the algorithms are proved by the Lyapunov theory.

Keywords: Self-tuning control, discrete-time systems, generalized minimum variance control, and sliding-mode control, linear control systems.

1. INTRODUCTION

Astrom and Wittenmark (Astrom and Wittenmark, 1973) developed and studied the convergence of the implicit self-tuning controller in a stochastic setting. The stability of self-tuning algorithms for Model Reference Adaptive Systems (MRAS) have been studied for the strictly positive real model by Landau (Landau, 1980)(Landau, 1982). Latter on, Johansson (Johansson, 1986) studied the stability of MRAS for a minimum phase system, using Lyapunov theory. Global convergence for a class of adaptive control algorithms applied to discrete-time single-input single-output (SISO) and multi-input multi-output (MIMO) linear systems were studied for the minimum variance criterion in a seminal paper by Goodwin, et al. (Goodwin *et al.*, 1980). However in these approaches the considered system should be minimum phase and the extension to consider the measurement noise may be di cult.

Extending the results of Astrom (Astrom and Wittenmark, 1973), Clarke and Gawthrop (Clarke *et al.*, 1975) proposed the Generalized Minimum Variance Control (GMVC) for non-minimum phase systems, using a cost function which incorporates system input and set-point variation. For

[1] Supported by the COE Grant in Aid of MEXT, Japan

the case of the unknown system parameters, the unknown parameters are estimated using a recursive least-squares algorithm. Latter, in (Clarke *et al.*, 1979) the convergence of the closed-loop system is analyzed using the positive real condition. In (Gawthrop, 1980) some stability analysis are given based on the notion of dissipative systems and conicity properties. However a complete global stability proof of self-tuning control for non-minimum phase systems have not been yet studied.

Sliding mode control (SMC) based on the variable structure systems (VSS), in the continuous-time case, is not robust when uncertainty excess the bound assumed in the design. Slotine (Slotine and Li, 1964) combined variable structure and adaptive control to solve this problem. Furuta (Furuta, 1993*a*) presents a discrete-time VSS type method for the case where systems parameters are unknown. The VSS is designed based on minimum variance control (MVC) or generalized minimum variance control (GMVC) using recursive parameter estimation. Extending (Furuta, 1993*a*), in (Furuta, 1993*b*) a designed parameter is introduced in the control law while maintaining the use of VSS. The stability of self-tuning control based on the certainty equivalence principle has been studied in (Morse, 1992). This approach is said implicit self-tuning control. However parameters are not identified accurately in the closed loop, and the stability is not assured based on the certainty equivalence principle.

In this paper, the stability of two types of implicit self-tuning controllers for discrete time minimum and non-minimum phase plants, when the system parameters are unknown, is proved. One is the combination of the generalized minimum variance control and identification of control parameter recursively (REGMVC), which has been used in many self-tuning controllers. The stability of the overall adaptive system is proved in this paper, although the parameters are not assured to converge to the true values. The other one (REGMVC-β) considers delay in control input. Stability of the algorithm is proved by the Lyapunov theory. It is not necessary to use VSS nor any additional condition to ensure closed loop system stability for the algorithms studied in this paper. The stability of the closed-loop system is proved in straight forward way in comparison with Goodwin, et al. (Goodwin *et al.*, 1980), and may be extended to the case including the measurement noise (Patete *et al.*, n.d.). This paper consider the non-minimum phase systems contrary to (Goodwin *et al.*, 1980) paper.

The paper is organized as follows; in section 2, the generalized minimum variance criterion is presented. Section 3 deals with parametric uncertainties using self-tuning control based on generalized minimum variance criterion. A simulate examples is given in section 4. Concluding remarks are in 5.

2. GENERALIZED MINIMUM VARIANCE CRITERION

This paper considers a single-input single-output (SISO) time-invariant system. The representation of the plant with input u_k and output y_k is

$$A(z^{-1})y_k = z^{-d}B(z^{-1})u_k \tag{1}$$

where $A(z^{-1})$ and $B(z^{-1})$ have no common factor and z denotes the time shift operator $z^{-t}y_k = y_{k-t}$. In the Laplace transformation, $z = e^{sT_0}$ where T_0 is the sample period (for simplicity, and without loss of generality, we may assume $T_0 = 1$).

The polynomials $A(z^{-1})$ and $B(z^{-1})$ are assumed to be known, and represented as:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_n z^{-n}$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + ... + b_m z^{-m}$$

where $b_0 \neq 0$ and delay step d, is also assumed to be known.

The objective of the control is to minimize the variance of the controlled variables s_{k+d}, that is defined in the deterministic case as

$$s_{k+d} = C(z^{-1})(y_{k+d} - r_{k+d}) + Q(z^{-1})u_k \tag{2}$$

The polynomials

$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + ... + c_n z^{-n}$$
$$Q(z^{-1}) = q_0(1 - z^{-1})$$

are to be designed, r_k is the reference signal, and the error signal e_k is defined as $e_k = y_k - r_k$.

The idea is similar to the discrete time sliding mode control, see (Xinghuo and Jian-Xin, 1992) and (Zinober, 1994). In the case of Goodwin, et al. (Goodwin *et al.*, 1980), $Q(z^{-1})$ is not considered and $C(z^{-1})$ is chosen as $C(z^{-1}) = 1$.

The polynomial $C(z^{-1})$ is Schur, hence the error signal will vanish if (2) is kept to zero. The polynomial $C(z^{-1})$ may be determined by assigning all characteristic roots inside the unit disk of z-plane.

Equation (2) is rewritten as:

$$s_{k+d} = G(z^{-1})u_k + F(z^{-1})y_k - C(z^{-1})r_{k+d} \tag{3}$$

where the polynomial $G(z^{-1})$ is defined as $G(z^{-1}) = E(z^{-1})B(z^{-1}) + Q(z^{-1})$, and polynomials $E(z^{-1})$ and $F(z^{-1})$ satisfy the equality,

$$C(z^{-1}) \doteq A(z^{-1})E(z^{-1}) + z^{-d}F(z^{-1}) \tag{4}$$