IMPROVING THE INTERSAMPLE BEHAVIOR BY USING A MULTIESTIMATION SCHEME WITH MULTIRATE SAMPLING

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Abstract: A multiestimation adaptive control scheme for linear time-invariant (LTI) continuous-time plant with unknown parameters is presented. The set of discrete adaptive models is calculated from a different combination of the correcting gain β in a fractional order hold (FROH) and the set of gains to reconstruct the plant input under multirate sampling with fast input sampling. The reference output is given by a continuous transfer function to evaluate the continuous tracking error of all the possible discrete models. Then the scheme selects online the model with the best continuous tracking performance. The estimated discrete unstable zeros are avoided through an appropriate design of the multirate gains so that the reference model might be freely chosen with no zeros constrains. A least-squares algorithm is used to estimate the plant parameters. However, only the active model is updated by using a least squares algorithm. The remaining possible models are updated by first calculating an estimated continuous-time transfer function, which results to be identical for all the models while their discretized versions are distinct in general.

Keywords: Adaptive control, Fractional Order Hold, Multirate sampling, Multiestimation, Supervised switching

1. INTRODUCTION

It is well-known that the unstable either continuous or discrete plant zeros should be transmitted to the reference model in a model matching problem (Aström and Wittenmark, 1990). In the context of discrete-time controllers acting on continuous-time plants, an appropriate choice of the correcting gain β of a FROH (potentially including zero-order holds, ZOH, for $\beta = 0$ and first-order holds, FOH, for $\beta = 1$) as well as the sampling period can locate some of the discretization zeros in the stable zone (Bilbao-Guillerna et al., 2005; Ishitobi, 1996; Liang and Ishitobi, 2005). However, this is not always possible because of the presence of unstable continuous-time zeros or because of the required range of the sampling period which can instabilize either the discretization or the intrinsic zeros. A solution of general applicability to avoid or circumvent this drawback is the use of multirate sampling techniques. A good selection of the multirate gains may make the estimated discrete zeros stable, (Alonso-Quesada and de la Sen, 2006; De la Sen and Bárcena, 2007; Moore et al., 1993; Morris and Neuman, 1981). However, the use of these techniques introduces a disadvantage that should be taken into account. Although the tracking of the desired reference can be achieved at sampling instants by the control law, the behavior of the output during the inter-sample period may not be suitable enough. This behavior depends on the choice of β, the sampling period and the reconstruction method used to generate the continuous plant input from the computed control sequence at sampling instants.

The main objective of this paper is to improve the inter-sample behavior by an appropriate selection of the gain β and the multirate gains through a fully freely chosen reference model even when the continuous plant possesses unstable zeros. In order to achieve this objective, we introduce a parallel multiestimation scheme, (Bilbao-Guillerna et al., 2005; Narendra and Balakrishnan, 1994 and 1997). The various models of this scheme are obtained via different values of the gain β in the FROH and different multirate gains. Since the plant parameters are unknown they have to be estimated and the models composing the multiestimation scheme are time-varying. *The main novelty of this paper compared with previous background work (Alonso-Quesada and de la Sen 2006, De la Sen and Alonso Quesada, 2007) is that the reference output is supplied by a stable continuous transfer function. Then the scheme is able to partly regulate the continuous-time tracking error while the controller is essentially discrete-time and operated by a FROH in general.* However, since the controller is designed to be discrete, it is necessary to obtain a discrete transfer function from the continuous-time reference one. In this way, each discretized plant model possesses a different discrete model which is obtained via discretization of the continuous reference model under a FROH with its associated gain β. As a result, each estimated model tends asymptotically to a different reference one. In (Alonso-Quesada and de la Sen, 2006), all of them converged asymptotically to the same reference model. The closed-loop performance is evaluated for all the possible discretized plant models by calculating their corresponding plant control signal and testing and monitoring its effect on an estimated continuous plant. Then, a performance index evaluates the continuous tracking performance of the estimated outputs related to the reference ones and the switching scheme selects the one with the lowest value. The active model currently in operation is used to online parameterize a discrete controller for matching the corresponding discrete reference model. A minimum residence time between consecutive switches is required for closed-loop stability purposes, (Aström and Wittenmark, 1990; Narendra and Balakrishnan, 1994 and 1997). Finally, some simulations will be displayed to show the effect of the proposed scheme.

2. DICRETE TRANSFER FUNCTION

Since the controller is discrete-time and the plant is continuous-time, we need to generate a continuous-time signal from the discrete control input, before injecting it to the plant. Two different reconstruction methods are considered in order to generate such an input. One method is governed by the large sampling period while the other one is governed by the small sampling period. The continuous-time plant is defined by the following state-space equations:

$$\dot{x}(t) = Ax(t) + bu(t); \quad y(t) = c^T x(t) \tag{1}$$

where $u(t)$ and $y(t)$ are, respectively, the input and output signals, $x(t) \in \Re^n$ denotes the state vector and A, b and c^T are constant matrix and vectors of appropriate dimensions.

2.1 Input Reconstruction Method I (Ruled by the large sampling period T)

The plant input is generated by the following equation:

$$u(t)=\alpha_j\left\{u_k+\beta\frac{u_k-u_{k-1}}{T}(t-kT)\right\} \tag{2}$$

for $t\in\left[kT+(j-1)T^{'},kT+jT^{'}\right)$ and $j\in\bar{N}\equiv\{1,2,...,N\}$, where u_k is the input signal at $t=kT$ and $\beta\in[-1,1]$ is the FROH correcting gain. T is the large sampling period associated with slow sampling rate of the output and $T^{'}=T/N$ is the small sampling period associated with the fast sampling rate of the input. In other words, the large sampling period is divided in N equal subperiods in order to generate the multirate input. It is possible to ensure the stability of the zeros of all the discretized plant models which relate the input and output sequences defined over the sampling period T by an appropriate choice of the multirate gains α_j since the discretized plant zeros are parameterized by such gains. The discrete transfer function is

$$H_\beta(z)=B_\beta(z)/A_\beta(z) \tag{3}$$

The denominator of the transfer function does not depend on the choice of the gains α_j and it can be calculated as:

$$A_\beta(z)=\begin{cases}Det\left(zI_n-\psi^N\right)=z^n+\sum_{\ell=1}^{n}a_\ell z^{n-\ell} & if\,\beta=0\\ zDet\left(zI_n-\psi^N\right)=z^{n+1}+\sum_{\ell=1}^{n}a_\ell z^{n-\ell+1} & if\,\beta\neq0\end{cases} \tag{4}$$

The numerator can be written as:

$$B_\beta(z)=\begin{cases}c^TAdj\left(zI_n-\psi^N\right)C_\Delta g=\sum_{\ell=1}^{n}b_\ell^{(0)}z^{n-\ell} & if\,\beta=0\\ c^TAdj\left(zI_n-\psi^N\right)\left(zC_\Delta-\beta C_\Delta^{'}\right)g=\sum_{\ell=1}^{n+1}b_\ell^{(\beta)}z^{n-\ell+1} & if\,\beta\neq0\end{cases} \tag{5}$$

with
$$C_\Delta=\left[\psi^{N-1}\Delta_1,...,\psi\Delta_{N-1},\Delta_N\right]\in\Re^{n\times N}$$
$$C_\Delta^{'}=\left[\psi^{N-1}\Delta_1^{'},...,\psi\Delta_{N-1}^{'},\Delta_N^{'}\right]\in\Re^{n\times N}$$
$$\Delta_j=\left(1+\frac{j-1}{N}\beta\right)\Gamma+\frac{\beta}{T}\Gamma^{'}\in\Re^{n\times1};\Delta_j^{'}=\frac{j-1}{N}\Gamma+\frac{1}{T}\Gamma^{'}\in\Re^{n\times1}$$
$$\Gamma=\int_0^{T^{'}}\phi(T^{'}-s)Bds\in\Re^{n\times1};\ \Gamma^{'}=\int_0^{T^{'}}\phi(T^{'}-s)Bsds\in\Re^{n\times1},$$
$$\psi^j=\phi\left(\frac{j}{N}T\right)=e^{j\frac{AT}{N}}\ \text{and}\ g^T=[\alpha_1,\alpha_2,...,\alpha_N]$$

with $Adj(.)$ and $Det(.)$ denoting, respectively, the adjoint matrix and the determinant of the square matrix (.) and I_n denoting the n-th order identity matrix. g is the vector of multirate gains with

$$\deg(A_\beta)=\deg(B_\beta)+1=\begin{cases}n & if\,\beta=0\\ n+1 & if\,\beta\neq0\end{cases} \tag{6}$$

The denominator can be rewritten as:

$$B_\beta(z)=\begin{cases}\sum_{\ell=1}^{n}\left(\sum_{j=1}^{N}b_{j,\ell}^{(0)}\alpha_j\right)z^{n-\ell}=\sum_{\ell=1}^{n}b_\ell^{(0)}z^{n-\ell} & if\,\beta=0\\ \sum_{\ell=1}^{n+1}\left(\sum_{j=1}^{N}b_{j,\ell}^{(\beta)}\alpha_j\right)z^{n-\ell+1}=\sum_{\ell=1}^{n+1}b_\ell^{(\beta)}z^{n-\ell+1} & if\,\beta\neq0\end{cases} \tag{7}$$

The coefficients $b_{j,\ell}$ depend on the parameters of the continuous time plant, the large sampling period T and the correcting gain β of the FROH considered in the discretization process. The value of the vector of multirate gains is relevant to stabilize the discrete plant zeros by appropriate choice of its components. Note that if we choose $\alpha_j=1$ for all $j\in\bar{N}$, then this reconstruction method becomes the common one obtained with a FROH without multirate sampling working at the large single sampling period T.

2.2 Input Reconstruction Method II (Ruled by the small sampling period T')

In this method, the plant input is governed with the fast sampling and generated by the following equation:

$$u(t)=u_k^{(j)}+\beta\frac{N}{T}\left(u_k^{(j)}-u_k^{(j-1)}\right)\left(t-\left(k+\frac{j-1}{N}\right)T\right) \tag{8}$$

for $t\in\left[kT+(j-1)T^{'},kT+jT^{'}\right)$ and $j\in\bar{N}$, where

$u_k^{(j)}:=u(kT+(j-1)T^{'})=\alpha_ju_k$ and $u_k^{(0)}:=u(kT-T^{'})=\alpha_Nu_{k-1}$

The denominator of the transfer function is identical to that obtained in (4), while the numerator is

$$B_\beta(z)=\begin{cases}B_0^T(z)g & if\,\beta=0\\ z\left[B_0^T(z)g+\frac{\beta N}{T}B_1(z)g'\right]-\frac{\beta N}{T}\alpha_NB_{1,1}(z) & if\,\beta\neq0\end{cases} \tag{9}$$

where, $B_0^T(z)=\left[B_{0,1}(z),B_{0,2}(z),...,B_{0,N}(z)\right]$;

$B_1^T(z)=\left[B_{1,1}(z),B_{1,2}(z),...,B_{1,N}(z)\right]$

with $B_{0,j}(z)=C^TAdj\left(zI_n-\psi^N\right)\psi^{N-j}\Gamma$;

$B_{1,j}(z)=C^TAdj\left(zI_n-\psi^N\right)\psi^{N-j}\Gamma^{'}$ and

$g'^T=\left[\alpha_1,\alpha_2-\alpha_1,...,\alpha_N-\alpha_{N-1}\right]$

Now the multirate is ruled by the fast sampling rate because it is necessary to know the value of the input at the fast sampling instants to generate the continuous-time plant input. Note that if $\beta=0$ both methods lead to the same transfer function.

2.3 Compact Representation

The discretized plant model can be described in a compact and clear way as

$$y_k=\begin{cases}-\sum_{\ell=1}^{n}a_\ell y_{k-\ell}+\sum_{\ell=1}^{n}\sum_{j=1}^{N}b_{j,\ell}^{(0)}\alpha_ju_{k-\ell}=\theta^{(0)T}\varphi_{k-1}^{(0)} & if\ \beta=0\\ -\sum_{\ell=1}^{n}a_\ell y_{k-\ell}+\sum_{\ell=1}^{n+1}\sum_{j=1}^{N}b_{j,\ell}^{(\beta)}\alpha_ju_{k-\ell}=\theta^{(\beta)T}\varphi_{k-1}^{(\beta)} & if\ \beta\neq0\end{cases} \tag{10}$$

where $\theta_\beta=\left[\theta_a^T\ \theta_{b,1}^T\ \theta_{b,2}^T\ ...\ \theta_{b,n+1}^T\right]^T;\theta_0=\left[\theta_a^T\ \theta_{b,1}^T\ \theta_{b,2}^T\ ...\ \theta_{b,n}^T\right]^T$

$$\varphi_{\beta,k}=[\varphi_y^T\ \varphi_{u,1}^T\ \varphi_{u,2}^T\ ...\ \varphi_{u,n+1}^T]^T;\varphi_{0,k}=[\varphi_y^T\ \varphi_{u,1}^T\ \varphi_{u,2}^T\ ...\ \varphi_{u,n}^T]^T;$$
$$\theta_a=[-a_1-a_2\ ...-a_n]^T;\theta_{b,\ell}=\left[b_{\ell,1}\ b_{\ell,2}\ ...\ b_{\ell,N}\right]^T$$
$$\varphi_y=[y_{k-1}\ y_{k-2}\ ...\ y_{k-n}]^T;\varphi_{u,\ell}=[\alpha_1u_{k-\ell}\ \alpha_2u_{k-\ell}\ ...\ \alpha_Nu_{k-\ell}]^T$$

The above notation will be then useful in order to formulate properly the estimation scheme with the given expanded regressor. The coefficients of the numerator of the discrete transfer function can be rewritten as

$$v_\beta=M_\beta g \tag{11}$$

where

$$M_\beta=\begin{bmatrix}b_{1,1}^{(\beta)}\ b_{2,2}^{(\beta)}\cdots b_{N,1}^{(\beta)}\\ b_{1,2}^{(\beta)}\ b_{2,2}^{(\beta)}\cdots b_{N,2}^{(\beta)}\\ \vdots\ \ddots\\ b_{1,n+1}^{(\beta)}\ \cdots\ b_{N,n+1}^{(\beta)}\end{bmatrix}\ \text{and}\ v_\beta=\left[b_1\ b_2\ ...\ b_{n+1}\right]^T\ \text{if}\ \beta\neq0$$

$$M_0=\begin{bmatrix}b_{1,1}^{(0)}\ b_{2,1}^{(0)}\cdots b_{N,1}^{(0)}\\ \vdots\ \ddots\\ b_{1,n}^{(0)}\ \cdots\ b_{N,n}^{(0)}\end{bmatrix}\ \text{and}\ v_0=\left[b_1\ b_2\ ...\ b_n\right]^T\ \text{if}\ \beta=0 \tag{12}$$