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Research Paper

Flexibility and rigidity of cross-linked Straight Fibrils under axial motion constraints

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ABSTRACT

The Straight Fibrils are stiff rod-like filaments and play a significant role in cellular processes as structural stability and intracellular transport. Introducing a 3D mechanical model for the motion of braced cylindrical fibrils under axial motion constraint; we provide some mechanism and a graph theoretical model for fibril structures and give the characterization of the flexibility and the rigidity of this bar-and-joint spatial framework. The connectedness and the circuit of the bracing graph characterize the flexibility of these structures. In this paper, we focus on the kinematical properties of hierarchical levels of fibrils and evaluate the number of the bracing elements for the rigidity and its computational complexity. The presented model is a good characterization of the frameworks of bio-fibrils such as microtubules, cellulose, which inspired this work.

1. Introduction

The fibril network literature declares that the cross-linked fibril structure is complicated and consist of redundant connections (Blundell and Terentjev; Ha and Thirumalai, 1997; Ingber, 1998; Liljenström, 2003; Väkiparta et al., 2004). These redundancies are necessary the loading, the signal transporting, and the stability point of view. However, technical systems or the structures of the natural or social sciences have to be rigid and/or flexible some case at the same time (Csermely, 2006; Fletcher and Mullins, 2010; Fratzl, 2008; Gáspár and Csermely; Head et al., 2003; Jacobs et al., 2001). This paper characterizes the rigidity and mobility of rod-like fibrils in biological systems. The rod-like fibril structure with redundant bracing elements is safe if some of the bracing elements would collapse than the remainders make the structure rigid yet. Celluloses (Cosgrove, 2014; Gibson et al., 2010; Gibson, 2013; Liu et al., 2013; Park and Cosgrove, 2012; Saitoh et al., 2013; Svensson et al., 2010; Tanaka et al., 2012), Fibrin (Ferry, 1952), Collagens, Minerals, Microtubules, (Ahmadzadeh et al., 2015; Cheng. and Pinsky, 2013; Gardel et al., 2004; Genin et al., 2009; Gutjahr et al., 2006; Karsai et al., 2006; Kasza et al., 2010; Licup et al., 2015; Motte and Kaufman, 2013; Pelletier et al., 2003; Thorpe, 1983; Wang, 2006; Wood and Keech, 1960; Zhang et al., 2014; Zimmermann and Ritchie, 2015), Chitins (Ilnicka and Lukaszewicz, 2015; Prashanth and Tharanathan, 2007; Sachs et al., 2006; Sachs et al., 2008) self-assemble into thick, hierarchically ordered, stiff fibres through electrostatic and hydrophobic interactions,. The network stiffness becomes surprisingly insensitive to network concentration, demonstrate how a simple model for networks of elastic fibres can quantitatively account

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for the mechanics of reconstituted collagen networks. We provide a discrete model that characterizes the flexibility and rigidity of braced framework of the fibrils. The model gives a good characterization in the case when the Young modulus is high enough, the order of magnitude not less than 1 GPa for the fibres and the crosslinks also. The paper gives a model for further improvements. This model provides an input to other tests that take account of the exact mechanical properties of the fibrils, the cross-links, and the matrix. The mineral crystals in nano bone structure have a large aspect ratio. Hence, the direction of its displacement is the same as the macro fibrils that contain them and the micro fibril that connect them. Besides bone, other biological materials like shell, dentin, spider silk, wood, and chitin or the microstructural model of the axonal cytoskeleton (Ahmadzadeh et al., 2015) show similar structure despite their complex hierarchical structures. They are arranged in grid position in cross-section view, cross-linked by braces with multiplicity, which consist of similar material. The lower bound of the distance between any two cylinders in the packing is the thickness of the cylinder. In the case of dense packing of the cylinders (in Bundle or Layer structure), they can move only in its axial direction; the adjacent cylinders obstruct the lateral motions. Notwithstanding, if the axial motion constraints are not satisfied in a cross-linked fibril structure than our model gives a necessary condition for the rigidity of the bar–joint structure. If the SFF of the a cross-linked fibril structure is not rigid then its bar–joint structure is not rigid. Hence, further cross-links have to be applying for the rigidity that is count by the Maxwell's rank condition for the further method that should take account of the exact mechanical properties of the fibrils, the cross-links, and the matrix.

1.1. *Bar-and-joint framework*

The bar–joint framework: One of the simplest structures in statics is the bar-and-joint framework, that consists of optimal bars connected by rotatable joints, i.e. the bar lengths and the bar and joint incidences must be preserved.

1.1.1. *The rigidity of bar-and-joint framework*

Firstly, we give a definition of the rigidity of the bar-and-joint framework.

Definition 1. A framework is rigid if any continuous motion of the joints that keeps the length of every bar fixed, also keeps the distance fixed between every pair of joints.

It is a preservation of distances the joints during any continuous motion of the joints.

1.1.2. *The infinitesimal rigidity of bar–joint framework*

The infinitesimal motion is a special case of virtual motion; that refers to a virtual change in the position such that the constraints remain satisfied (possible motions).

The rigid body motions refer to the trivial infinitesimal motion. The Definition 1 allows frameworks that have infinitesimal motions. In the statics for the rigid structure, the non-trivial infinitesimal motions of the joints have not permitted (Libonati et al., 2013; Nagy, 2001; Owen and Power, 2010; Power, 2014; Radics and Recski, 2002; Recski, 1989; Szymanski, 2014; Thomas et al., 2013).

Definition 2. A framework is infinitesimally rigid if it only has trivial infinitesimal motions.

We can see the central joints of the framework on Fig. 1. It has an infinitesimal motion up and down, perpendicularly to the plane of the structure. If a framework is infinitesimally rigid, we require first-order preservation of distances during the infinitesimal motions of all joints. We have to decide the rank of rigidity matrix of the framework.

Let (x_i, y_i, z_i) be the coordinates of the joint P_i of a bar–joint structure, where $(1 \le i \le n)$. A bar between the joints P_i and P_j determines the distance from P_i to P_j, there for it is constant, by differentiating its square, leads to the next equation:

$$(x_i - x_j)(\dot{x}_i - \dot{x}_j) + (y_i - y_j)(\dot{y}_i - \dot{y}_j) + (z_i - z_j)(\dot{z}_i - \dot{z}_j) = 0. \tag{1}$$

where, the velocity coordinates $\dot{x}_i$, $\dot{y}_i$, $\dot{z}_i$ are the varieties. Hence, if we use bars between joints, and the number of bars is e, then we get a system with e pieces of equations. The matrix representation of the equation system is the next:

$$Au = 0. \tag{2}$$

where u is the column vector of velocity, and A is an $e \times 3n$ rigidity matrix. In statics, rigidity does not even allow infinitesimal motions. In this case, the Eq. (2) has the trivial solution only (i.e. the rigid body like motions). The rigid body motion of the joint keeps fixed the distance between the pairs of the joint. If the joints of the framework have motions, that different from the rigid body motion, then the framework is not rigid. In this case the rank$(A) < 3n-6$. The framework is rigid if and only if the rank$(A) = 3n-6$, see: (Jordán et al., 2013; Maxwell, 1864; Nagy, 1994; Owen and Power, 2010; Power, 2014; Recski, 1989; Thomas et al., 2013; Watanabe and Nakamura, 1993). Maxwell (1864) gave this characterization but with his result, the time complexity of deciding the rigidity is $O(e^3)$.

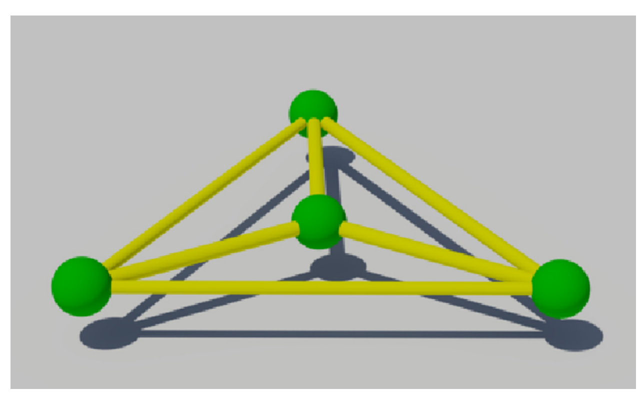

Fig. 1 – Infinitesimal motion: There are non-trivial infinitesimal motions of the central joints of the framework. The central points of its joints are in the same plane. The central joint can move infinitesimally into the normal direction of framework plane, at the beginning of this motion, no constraint restrict these motions. The length of the shorter bars change in second-order, the preservation of all distances is first order.