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Research Paper

Shape oscillations of elastic particles in shear flow

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ABSTRACT

Particle suspensions are common to biological fluid flows; for example, flow of red- and white-blood cells, and platelets. In medical technology, current and proposed methods for drug delivery use membrane-bounded liquid capsules for transport via the microcirculation. In this paper, we consider a 3D linear elastic particle inserted into a Newtonian fluid and investigate the time-dependent deformation using a numerical simulation. Specifically, a boundary element technique is used to investigate the motion and deformation of initially spherical or spheroidal particles in bounded linear shear flow. The resulting deformed shapes reveal a steady-state profile that exhibits a 'tank-treading' motion for initially spherical particles. Wall effects on particle trajectory are seen to include a modified Jeffrey's orbit for spheroidal inclusions with a period that varies inversely with the strength of the shear flow. Alternately, spheroidal inclusions may exhibit either a 'tumbling' or 'trembling' motion depending on the initial particle aspect ratio and the capillary number (i.e., ratio of fluid shear to elastic restoring force). We find for a capillary number of 0.1, a tumbling mode transitions to a trembling mode at an aspect ratio of 0.87 (approx.), while for a capillary number of 0.2, this transition takes place at a lower aspect ratio. These oscillatory modes are consistent with experimental observations involving similarly shaped vesicles and thus serves to validate the use of a simple elastic constitutive model to perform relevant physiological flow calculations.

1. Introduction

Suspensions of deformable particles (e.g., blood cells, platelets, liposomes) are common to biological fluid flows. While the inclusions, rigid or otherwise, affect fluid rheology, particle deformation may play a role in important physiological function. For example, the rolling deformation of leukocytes as occurs during cellular inflammatory response may enhance their adhesive interactions with inflamed tissue (Park et al., 2002; Subramaniam, 2012). Similarly, deformation of erythrocytes may trigger cellular release of ATP (Wan et al., 2011). Also, the membrane stability of polymer micro-gels that serve as drug delivery capsules could be influenced by their deformability (Oh et al., 2008)(D:/

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JournalITrack/Login/ELSE/JMBBM/1944/S100/Version1-30May2016205403/) – Oh et al., 2008. In short, particle deformation can affect physiological function.

Following the pioneering work of (Fröhlich and Sack, 1946), a number of computational methods have been developed to simulate the deformation and shape oscillations of biological cells and capsules modeled as liquid drops. Eggleton and Popel (1998) used the immersed boundary method (IBM) to simulate large deformations of red-blood cells in shear flow. Several variants including penalty IBM (Huang et al., 2012), implicit (Le et al., 2009), Lattice Boltzmann method (LBM) – IBM (Sui et al., 2008, 2010a), and front-tracking methods (Li and Sarkar, 2008) have been developed to study the shape oscillations of liquid-filled elastic capsules in unbounded shear flow and hyperelastic solids in cavity flow (Zhao et al., 2008). Some of these methods have been extended to include wall effects in bounded shear flow or Couette flow (Song et al., 2011), and all require the discretization of the Navier–Stokes or Lattice-Boltzmann equations over a fixed Cartesian grid. More recently, the Arbitrary Lagrangian Eulerian (ALE) finite-element (FEM) (Gao et al., 2011) and Eulerian finite-difference method (Sugiyama et al., 2011) were developed to study shape oscillations of a deformable particle modeled as a neo-Hookean solid in unbounded shear flow. Villone et al. (2015) used a 3D ALE finite element formulation to study the dynamic behavior of an incompressible neo-Hookean elastic prolate spheroid suspended in bounded or unbounded shear flow of a Newtonian fluid. Also, the boundary element method (BEM) has been used to compute the finite deformation of liquid drops (Kennedy et al., 1994), liquid capsules (Lac et al., 2004; Pozrikidis, 1995), and red-blood cells (Pozrikidis, 2003).

In the special case of uniaxial extension of a neo-Hookean solid subject to finite strain, the principal stress σ and principal stretch λ are related through a non-linear relation,

$$\sigma = \eta\left(\lambda - \frac{1}{\lambda^2}\right) \quad (1)$$

where η is the shear modulus and $\lambda = 1+\varepsilon$ (ε is the infinitesimal strain). For $\varepsilon \ll 1$, Eq. (1) reduces to

$$\sigma = 3\eta\varepsilon \quad (2)$$

which implies that the equivalent elastic modulus for an incompressible neo-Hookean solid is equivalent to 3η; identical to linear elasticity. Thus, elastomers – under conditions of infinitesimal strain – can be approximated using a linear elastic constitutive model.

The present study uses a BEM which is well-suited for the problem based on its ability to treat flow suspensions in bounded domains without the need for container discretization. The suspended particle (initially spherical or spheroidal) is modeled as a compressible, isotropic, homogeneous linear elastic solid. Specifically, the mobility and shape dynamics problems for deformable particles in bounded linear shear flow (Fig. 1) are computed using a mixed (both indirect and direct) boundary element method (Phan-Thien and Fan, 1995; Phan-Thien et al., 1992; Power and Miranda, 1987). The formulation uses the method of reflections to enforce no-slip, no-displacement boundary conditions at the wall (Blake and Chwang, 1974; Blake, 1971) and, further, assumes that the mobility, exterior, and interior problems may be de-coupled and solved sequentially. Specifically, during solution of the exterior problem for traction, the particle is considered rigid. The solution technique follows largely from, and is described in Phan-Thien and Fan (1996). Discoid-shaped cells such as erythrocytes may be modeled as oblate spheroids, while unstressed leukocytes are modeled as spheres. Leukocytes have previously been modeled as elastic solids (Gee and King, 2010), and the resulting deformed shapes were shown to be consistent with cell deformations that occur during the stop-and-go motion of leukocytes during selectin-mediated rolling (Evans et al., 2005). The current study considers the limiting case of vanishing Reynolds number (Re$\ll$1) which is representative of flow in the microcirculation. Additionally, the initial particle orientation is such that two of the three mutually orthogonal major and/or minor axes are contained on the shear plane. Initial orientations of the particle off of this plane have not been considered here.

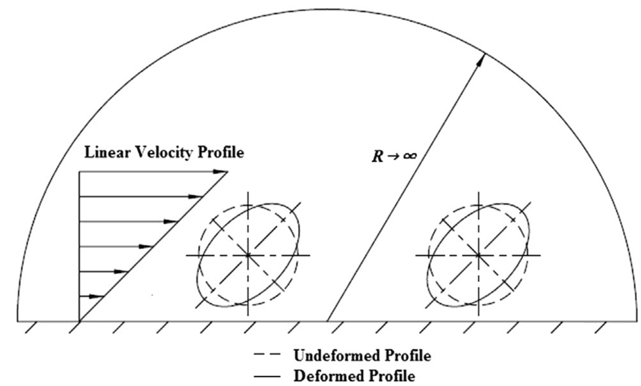

Fig. 1 – Elastically deformable particles in wall-bounded shear flow.

2. Theory

For vanishing Reynolds number, the Navier–Stokes equations simplify to a linear set known as Stokes equations,

$$-\nabla p + \mu\nabla^2 \boldsymbol{v} = \mathbf{0}, \quad \nabla \cdot \boldsymbol{v} = 0 \quad (3)$$

where $\boldsymbol{v}$, p are velocity and pressure, respectively. These fundamental equations of fluid mechanics express the balance of pressure and viscous forces, and mass conservation, respectively. Note, since the only body force considered here is due to gravity, it can be expressed as the gradient of a potential and combined with the pressure term. No-slip boundary conditions are enforced at the planar interface at all times. Immersed cells are subject to fluid shear and, possibly, contact stresses (Subramaniam et al., 2013). During cellular inflammatory response, rolling leukocytes are also subject to adhesive stresses as a result of transient receptor-ligand binding (King and Hammer, 2001). Small, infinitesimal strains associated with cell rolling may be described with the strain-displacement relation $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$ where $\boldsymbol{u}$ is displacement and $u_{i,j}$ denotes the partial derivative. Substituting this into Hooke's law relation for elastic isotropic response $T_{ij} = \lambda_p \varepsilon_{kk}\delta_{ij} + 2\eta\varepsilon_{ij}$, where η, λ_p are Lamé elastic constants, ε_{kk} denotes trace, and δ_{ij} is the Kronecker delta, yields the