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Research Paper

On the accuracy and fitting of transversely isotropic material models

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ABSTRACT

Fiber reinforced structures are central to the form and function of biological tissues. Hyperelastic, transversely isotropic material models are used widely in the modeling and simulation of such tissues. Many of the most widely used models involve strain energy functions that include one or both pseudo-invariants (I_4 or I_5) to incorporate energy stored in the fibers. In a previous study we showed that both of these invariants must be included in the strain energy function if the material model is to reduce correctly to the well-known framework of transversely isotropic linear elasticity in the limit of small deformations. Even with such a model, fitting of parameters is a challenge. Here, by evaluating the relative roles of I_4 and I_5 in the responses to simple loadings, we identify loading scenarios in which previous models accounting for only one of these invariants can be expected to provide accurate estimation of material response, and identify mechanical tests that have special utility for fitting of transversely isotropic constitutive models. Results provide guidance for fitting of transversely isotropic constitutive models and for interpretation of the predictions of these models.

1. Introduction

Fiber-reinforced structure is typical of many soft biological tissues, including skeletal muscle (Morrow et al., 2010), myocardium (Humphrey, 2002; Taber, 2004), brain stem (Ning et al., 2006), white matter (Feng et al., 2013), ligament and tendon (Dourte et al., 2008; Lake et al., 2010; Thomopoulos and Genin, 2012; Weiss et al., 1996). To understand the mechanical behavior of these soft biological tissues, reliable material models are needed. Here, we focus on transversely isotropic hyperelastic models for such tissues. The constitutive properties are described by a strain energy function ψ, which is a function of certain measures of deformation (Spencer, 1984), some of which (I_1, I_2, I_3) are

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Table 1 – A summary of incompressible, transversely isotropic, hyperelastic models.[a]

	$\psi_{isotropic}$	$\psi_{anisotropic}$	Volumetric term[b]
Spencer (1984)	$\mu_T tr(\boldsymbol{\varepsilon}^2)$	$2(\mu_L-\mu_T)\mathbf{A}\bullet\boldsymbol{\varepsilon}^2\bullet\mathbf{A}+\frac{1}{2}\beta(\mathbf{A}\bullet\boldsymbol{\varepsilon}\bullet\mathbf{A})^2$	$\frac{1}{2}\lambda(tr\varepsilon)^2$
Polignone and Horgan (1993)[c]	$\frac{\mu}{2}(I_1-3)$	$\frac{\mu}{2}a(I_5^2-2I_5+1)$	–
Weiss et al. (1996)	$C_1(\bar{I}_1-3)+C_2(\bar{I}_2-3)$	$C_3(exp(\bar{I}_4-3)-\bar{I}_4)$	–
Qiu and Pence (1997)	$\frac{\mu}{2}(I_1-3)$	$\frac{\mu\gamma}{2}(I_4-1)^2$	–
Taber (2004)	$b_1e^{b_2(I_1-3)}$	$b_3(\sqrt{I_4}-1)^m$	–
Merodio and Ogden (2005)	$\frac{\mu}{2}(I_1-3)$	$\frac{\mu\gamma}{2}(I_5-1)^2$	–
Horgan and Saccomandi (2005)[d]	$\frac{\mu}{2}(I_1-3)$	$\left\{\begin{array}{l} -\mu_I J_m^I\left((I_4-1)+J_m^I ln\left(1-\frac{I_4-1}{J_m^I}\right)\right) \\ -\frac{\mu_I}{2}J_m^{II}ln\left(1-\frac{(I_4-1)^2}{J_m^{II}}\right) \\ -\frac{\mu_{II}}{2}J_m^{III}ln\left(1-\frac{(I_5-1)^2}{J_m^{III}}\right)\end{array}\right.$	–
Schröder et al. (2005)	$\frac{\alpha_1 I_1}{J^{\frac{1}{3}}}+\frac{\alpha_2 I_2}{J^{\frac{1}{3}}}-\alpha_3 ln(J)+\alpha_4\left(I_3^{\alpha_5}+\frac{1}{I_3^{\alpha_5}}-2\right)$	$\alpha_6(I_5-I_1I_4+I_2)+\frac{\alpha_7 I_4^{\alpha_8}}{J^{\frac{1}{3}}}+\alpha_9(I_1I_4-I_5)+\alpha_{10}I_4^{\alpha_{11}}$	–
Lu and Zhang (2005)[e]	–	$\frac{1}{2}k_4(\beta_2-2)+k_2exp\left(c(\bar{\lambda}-1)^2\right)+\frac{1}{2}k_3(\beta_1-1)$	$\frac{k_1}{2}(J-1)^2$
Ning et al. (2006)	$C_{10}(\bar{I}_1-3)$	$\frac{\theta}{2}(\overline{I_4}-1)^2$	$\frac{1}{D}(J-1)^2$
Velardi et al. (2006)	$\frac{2\mu}{\alpha^2}(\lambda_1^\alpha+\lambda_2^\alpha+\lambda_3^\alpha-3)$	$\frac{2k\mu}{\alpha^2}\left(I_4^{\alpha/2}+I_4^{-\alpha/4}-3\right)$	$\lambda_1\lambda_2\lambda_3=1$
Gasser et al. (2006)	$\frac{\mu}{2}(\bar{I}_1-3)$	$\frac{k_1}{2k_2}\left[exp\left\{k_2\left[\kappa\bar{I}_1+(1-3\kappa)\bar{I}_4-1\right]^2\right\}-1\right]$	–
Chatelin et al. (2012)	$C_{10}(\bar{I}_1-3)+C_{01}(\bar{I}_2-3)$	$W_{fibers}^d(\bar{I}_4)$	$\frac{1}{2}KlnJ^2$
Feng et al. (2013)	$\frac{\mu}{2}(\bar{I}_1-3)$	$\frac{\mu\zeta}{2}(\overline{I_4}-1)^2+\frac{\mu\phi}{2}\overline{I_5^*}$	$\frac{\kappa}{2}(J-1)^2$
Destrade et al. (2015); Horgan and Murphy (2015); Murphy (2013)[d]	$\frac{\mu_T}{2}[\alpha(I_1-3)+(1-\alpha)(I_2-3)]$	$\left\{\begin{array}{l} \frac{\mu_T-\mu_L}{2}(2I_4-I_5-1)+\frac{E_L+\mu_T-4\mu_L}{8}(I_4-1)^2 \\ \frac{\mu_T-\mu_L}{2}(2I_4-I_5-1)+\frac{E_L+\mu_T-4\mu_L}{32}(I_5-1)^2 \\ \frac{\mu_T-\mu_L}{2}(2I_4-I_5-1)+\frac{E_L+\mu_T-4\mu_L}{16}(I_4-1)(I_5-1)\end{array}\right.$	–
Swedberg et al. (2014)	$\frac{\mu}{2}(I_1-3)$	$\frac{c_1}{2c_2}\left(exp\left(c_2(\lambda-1)^2\right)-1\right)$	[f]

[a] In the incompressible case the isochoric invariants $\overline{I_1},\overline{I_2},\overline{I_4},\overline{I_5}$ are effectively equal toI_1,I_2,I_4,I_5 . We keep the original form of each strain energy function.
[b] Explicit volumetric term is shown here if it is written out specifically in the original references.
[c] The invariant I_5 in the formulation corresponds to the definition of I_4 in this paper.
[d] Strain energy function is composed of one of the forms of $\psi_{anisotropic}$.
[e] The formulation used a multiplicative decomposition of the deformation that factor out the volumetric strain and fiber stretch. $\bar{\lambda}=J^{-1/3}\lambda$, $\beta_1=\frac{1}{\lambda^4}\mathbf{C}^2\mathbf{N}\otimes\mathbf{N}$ $\beta_2=\frac{\lambda}{J}trC-\frac{1}{J\lambda}\mathbf{C}^2\mathbf{N}\otimes\mathbf{N}$.
[f] $-\mu\ln\sqrt{I_3}+\frac{\kappa}{2}\left(ln\left(\frac{I_5-I_1I_4+I_2}{I_4^{2(m-v_0)}e^{-4m(\lambda-1)}}\right)\right)^2$.

invariant under arbitrary rotations and others of which (I_4,I_5) are invariant under rotations about the fiber axis. The most general strain energy function form for a transversely isotropic material contains all of the five invariants (Taber, 2004):

$$\psi=\psi(I_1,I_2,I_3,I_4,I_5). \tag{1}$$

It is common to separate the strain energy function into two parts: the strain energy of the isotropic base material $\psi_{isotropic}$ and the strain energy associated with the anisotropic fiber components $\psi_{anisotropic}$ (Feng et al., 2013; Horgan and Saccomandi, 2005; Merodio and Ogden, 2003a,b, 2005; Murphy, 2013; Pierce et al., 2013; Qiu and Pence, 1997; Swedberg et al., 2014):

$$\psi=\psi_{isotropic}(I_1,I_2,I_3)+\psi_{anisotropic}(I_4,I_5). \tag{2}$$

A broad range of forms have been proposed for ψ (Table 1). Most proposed $\psi_{anisotropic}$ terms are expressed as a function of only one invariant, either with I_4 or I_5. Polignone and Horgan (1993) originally proposed a general quadratic form in terms of I_4 for $\psi_{anisotropic}$. Following this idea, many studies have focused on the strain energy function with variations of the form. Notably, the $\psi_{anisotropic}$ term considering tension-only fibers (e.g. I_4-1) is of primary interest for many biological applications such as ligaments and tendon tissues (Horgan and Saccomandi, 2005; Murphy, 2013). We study here the quadratic form of Qiu and Pence (1997), $F(I_4)=\mu\gamma(I_4-1)^2/2$, which we term the $F(I_4)$ model in which μ is a modulus that also appears in the isotropic part of ψ and γ is a dimensionless scaling factor. To study the effect of I_5, we evaluate the second term of the model of Feng et al. (2013), $G\left(\overline{I_5^*}\right)=\mu\phi\overline{I_5^*}/2$, in which ϕ is a dimensionless scaling factor and $\overline{I_5^*}=\bar{I}_5-\overline{I_4}^2$, where the overbar indicates a variable related to the distortional component of the deformation gradient, as described in detail in Section 2. We term this the $G(I_5)$ model, and note that many other forms for the role of I_5 have been proposed (Horgan and Saccomandi, 2005; Merodio and Ogden, 2005; Murphy, 2013).

Both Feng et al. (2013) and Murphy (2013) noted that both anisotropic invariants I_4 and I_5 are needed in the strain energy function to correctly describe tensile and shear