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Research Paper

One-dimensional nonlinear elastodynamic models and their local conservation laws with applications to biological membranes

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ABSTRACT

The framework of incompressible nonlinear hyperelasticity and viscoelasticity is applied to the derivation of one-dimensional models of nonlinear wave propagation in fiber-reinforced elastic solids. Equivalence transformations are used to simplify the resulting wave equations and to reduce the number of parameters. Local conservation laws and global conserved quantities of the models are systematically computed and discussed, along with other related mathematical properties. Sample numerical solutions are presented. The models considered in the paper are appropriate for the mathematical description of certain aspects of the behavior of biological membranes and similar structures.

1. Introduction

Biological membranes are thin soft biological structures that play vital roles in a living organism (Humphrey, 1998), and optimize their shape and function in response to stimuli from the environment (Badir et al., 2013; Checa et al., 2011). With their thickness rarely exceeding a few millimeters, biomembranes are able to withstand large physiological loads. Typical examples of biomembranes include the skin (Zöllner et al., 2012), the mucous membrane lining the air–organ interfaces of the respiratory and digestive systems (Li et al., 2011), the fetal membrane (Joyce et al., 2009), the tympanic membrane (Fay et al., 2005), and the heart valve membranes (Rabbah et al., 2013).

The membrane of a biological cell involves an assembly of filaments linked together as a part of a network, or associated with the cell membrane to build a two-dimensional thin sheet. Two-dimensional biological networks may be wrapped around a cell as its wall, or attached to its plasma or nuclear membrane. Structural elements of biological cells are soft and responsible for the large deformability and easy motion of the cell, contrary to majority of the engineered man-made thin structural materials used in sheet industries. The mechanics of biological membranes is clearly related to the network architecture and the elasticity of the filaments. Mechanical models for cells can be derived using either micro/nanostructural or continuum approaches, as explained in detail, for

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example, in Assidi et al. (2011) and references therein. Although the continuum approach is more straightforward, the identification of the continuum behavior of a membrane is challenging, as the membrane may be highly anisotropic due to unequal chain lengths and properties of the threads. The constitution of biomembranes as bilayers entails a rather floppy behavior, with bending as the dominant deformation mode in comparison to stretching (Boal, 2012).

The complex mechanical behavior of soft biological tissues results from the deformations and interactions of the constituent phases, including collagen, elastin, muscular, and matrix components, such as proteoglycans. Collagen fibre-rich tissues are classically modeled as composite materials made of one or several families of collagen fibers immersed into a very soft isotropic solid matrix composed mainly of proteoglycans. The preferred fibre alignment is described by a structural tensor entering the strain energy function (Boehler, 1978; Spencer, 1984). Elastin fibers stretch out at low mechanical strains, while the wavy collagen fibers uncrimp without a marked contribution to the overall skin stiffness (Limbert and Taylor, 2002). At higher strains, the stretched cross-linked collagen network carries most of the load up to the characteristic strain locking.

Due to the complex multi-scale hierarchical nature of biological tissues and the resulting difficulties in their experimental characterization, the development of structural models has been limited to favor continuum-based phenomenological hyperelastic and hyper-viscoelastic models (Hurschler et al., 1997; Holzapfel and Ogden, 2009; Criscione et al., 2003; Sacks, 2000). The traditional approach to formulate constitutive laws for biological soft tissues relies on invariant formulations which postulate the existence of a strain energy function depending on a set of tensorial invariants of a certain strain measure. Tensor invariants are selected that characterize the particular deformation modes reproducing real deformations of the tissue. An additive split of the fibre and matrix strain energies is assumed in such phenomenological models that accordingly decouple fibre and matrix effects (Holzapfel et al., 2000; Humphrey, 2003; Humphrey and Yin, 1987; Limbert and Taylor, 2002).

Experimental results reveal the insufficiency of elastic models due to their rough approximation of the actual response, since they ignore the time-dependent behavior of tissues (Prevost et al., 2011; Marchesseau et al., 2010). To address this deficiency, viscoelastic models are used (Fung, 1993; Roylance, 2001). Time-dependent responses of soft biological tissues have been analyzed through monotonic tensile tests at various strain rates and through creep tests (Arumugam et al., 1994; Pioletti et al., 1996; Yanjun et al., 2001; Shergold et al., 2006; Kettaneh et al., 2007). A large number of phenomenological constitutive models have been developed to simulate the experimentally observed anisotropic and time-dependent biomembrane response. The anisotropy is modeled by introducing structural tensors into the constitutive models, as illustrated in, e.g., Humphrey and Yin (1987), Ehret and Itskov (2007), Peña et al. (2011), and Maher et al. (2012). Viscous effects can be modeled using a viscous potential function (Germain, 1973; Pioletti and Rakotomanana, 2000; Roan and Vemaganti, 2011).

Wave propagation in soft biological materials has received considerable attention due to its importance for imaging techniques, which aim at implicit measurements of mechanical properties or visualization of organs (Valdez and Balachandran, 2013). In particular, tissue stiffness measurements can be performed in vivo, through the measurement of shear wave propagation speeds (Sarvazyan et al., 1998; Sandrin et al., 2003; Rouze et al., 2013). The development of accurate models for ultrasound propagation in soft tissues requires the consideration of nonlinear effects in wave propagation, due to the large amplitudes of the acoustic waves. Taking nonlinear effects into account is beneficial for modern ultrasound scanners that employ tissue harmonic imaging, since it provides images with improved clarity and contrast. The attenuation and dispersion of waves, as well as the wave speed, are essential parameters determining the depth reached by the waves and the quality of images. Due to the presence of constituents with viscous properties, soft tissues are absorbing at ultrasonic frequencies with the absorption following a frequency power law. In the context of nonlinear wave propagation, an accurate model of acoustic absorption is of particular importance as the generation of higher frequency harmonics due to nonlinear effects is balanced with their absorption. Furthermore, since soft biological tissues such as biomembranes contain different constituents, including water, their wave propagation characteristics, such as the sound speed and density, are weakly heterogeneous, with variations between the different types of soft tissue of the order of 5% (Krouskop et al., 1987).

The assessment of viscoelastic properties of soft tissues has raised a growing interest in the field of medical imaging in the last two decades, due to the fact that the measurements of local changes of stiffness can be used to detect pathologies. Methods related to dynamic elastography (Krouskop et al., 1987; Lerner et al., 1988; Yamakoshi et al., 1990), such as sonoelastography (Parker and Lerner, 1992; Levinson et al., 1995) or transient elastography (Bercoff et al., 2003; Sandrin et al., 2003), can be used to determine elastic properties of soft biological tissues. Beyond the estimate of second order elastic moduli, the quantification of the nonlinear, anisotropic and viscoelastic effects in soft solids (Catheline et al., 2003, 2004; Bercoff et al., 2004) is an important task that transient elastography is able to address, since the latter images, in real time, the transient propagation of shear waves. Based on the propagation of mechanical shear waves in tissues, diverse elastography techniques have the capability to quantitatively estimate the shear modulus of tissues, in a noninvasive manner (Bercoff et al., 2004; Palmeri et al., 2008; Mitri et al., 2011; Orescanin et al., 2010; Vappou et al., 2009; Hah et al., 2010).

The determination of elasticity model parameters for biological membranes is more involved in comparison to isotropic tissues, due to the occurrence of additional parameters associated with the fibrous microstructure. Initial stresses and/or strains are naturally present in soft biological tissues such as veins, arteries, skin, muscles, ligaments and tendons; for instance, skin is in a state of natural tension. The initial deformation introduces additional effective anisotropy into the wave propagation equations. The anisotropy of the deformation pattern due to either an initial state of finite deformation or to the fibrous microstructure is an important issue for various reasons: the wave speeds in biological materials are directionally dependent, and depend on the level and distribution of the existing deformation, onto which displacements associated with wave propagation are superimposed.

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