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Research Paper

Relaxed incremental variational approach for the modeling of damage-induced stress hysteresis in arterial walls

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ABSTRACT

In this paper, a three-dimensional relaxed incremental variational damage model is proposed, which enables the description of complex softening hysteresis as observed in supra-physiologically loaded arterial tissues, and which thereby avoids a loss of convexity of the underlying formulation. The proposed model extends the relaxed formulation of Balzani and Ortiz [2012. Relaxed incremental variational formulation for damage at large strains with application to fiber-reinforced materials and materials with truss-like microstructures. Int. J. Numer. Methods Eng. 92, 551–570], such that the typical stress-hysteresis observed in arterial tissues under cyclic loading can be described. This is mainly achieved by constructing a modified one-dimensional model accounting for cyclic loading in the individual fiber direction and numerically homogenizing the response taking into account a fiber orientation distribution function. A new solution strategy for the identification of the convexified stress potential is proposed based on an evolutionary algorithm which leads to an improved robustness compared to solely Newton-based optimization schemes. In order to enable an efficient adjustment of the new model to experimentally observed softening hysteresis, an adjustment scheme using a surrogate model is proposed. Therewith, the relaxed formulation is adjusted to experimental data in the supra-physiological domain of the media and adventitia of a human carotid artery. The performance of the model is then demonstrated in a finite element example of an overstretched artery. Although here three-dimensional thick-walled atherosclerotic arteries are considered, it is emphasized that the formulation can also directly be applied to thin-walled simulations of arteries using shell elements or other fiber-reinforced biomembranes.

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1. Introduction

In soft biological tissues a stress-softening behavior is observed at the macroscale when cyclically loading the tissue at supra-physiological load levels. This softening is assumed to be mostly induced by damage at the microscale evolving with increasing load. This is partially even desired, e.g. in arterial tissues, where this softening is exploited during a balloon angioplasty to increase the blood lumen of an atherosclerotic artery. Therefore, with the aim to optimize such clinical treatment methods the modeling of damage in soft biological tissues is subject of current research. From the modeling viewpoint, already Kachanov (1958) introduced the $(1-D)$-approach, which is commonly used in continuum damage mechanics (CDM). Later, Simo (1987) extended this to the finite strain framework, see also Miehe (1995). Various authors applied CDM to the modeling of soft biological tissues, while preceding the $(1-D)$-term to different parts of the overall strain-energy, see e.g. Balzani et al. (2006, 2012), Rodríguez et al (2006), Peña (2011, 2014), Gasser (2011), Saez et al. (2012), Forsell et al. (2013), Famaey et al. (2013) or Schmidt et al. (2014). Herein the anisotropic character of the damage is accounted for through scalar-valued damage variables D, which are associated to those strain-energy densities linked with the collagen fibers. Hokanson and Yazdani (1997) chose a different approach to model anisotropic damage in arteries by using a fourth order damage tensor. A further alternative is the inclusion of energy limiters into anisotropic strain-energy functions, see Volokh (2008, 2011). Ehret and Itskov (2009) took into account an evolution of structural tensors to model anisotropic damage of soft biological tissues. Moreover, also the framework of pseudo-elasticity may be used for damage modeling, see e.g. Ogden and Roxburgh (1999a), Ogden and Roxburgh (1999b), Dorfmann and Ogden (2004) and more recently Naumann and Ihlemann (2015). Thereby, an additional term provides that a change of the damage variable does not alter the value of the overall strain-energy. In Peña and Doblaré (2009), Weisbecker et al. (2012) and Pierce et al. (2015) the above approach is used to describe damage in soft biological tissues. Rickaby and Scott (2013), in contrast to that, model softening of biological tissues by preceding a softening function to the transversely isotropic part of the stress, whereby the difference between the current and the maximum stretch is accounted for in primary loading and the difference between the current and the maximum strain-energy is considered in unloading and reloading. A similar approach is followed by Rebouah and Chagnon (2014) to describe stress-softening of soft tissues, namely by preceding an evolution function to the fiber energy, which includes the difference between the current and the maximum strain. While some of the above continuum approaches are to some extent micromechanically motivated (Rodríguez et al, 2006; Gasser, 2011; Schmidt et al., 2014), a multiscale approach to model collagen fibril damage under consideration of the micro and the nanoscale is given in Marino and Vairo (2014). The above mentioned models in principle enable numerical damage computations in arteries within the framework of the finite element method. However, when it comes to softening the numerical solution may suffer from a loss of ellipticity of the underlying partial differential equation and mesh-dependent solutions might be obtained. To avoid this problem, e.g. Kuhl and Ramm (1999) or Dmitrievic and Hackl (2008) used gradient-enhanced damage models in a small strain framework. Steinmann (1999) was one of the first to propose a gradient-enhanced model for the geometrically nonlinear case by introducing a non-local strain-energy density as an additional primary variable. Waffenschmidt et al. (2014) formulated a large strain continuum damage model for fiber-reinforced materials by obeying the concept from Dmitrievic and Hackl (2008) and considering a split of the strain-energy into a local and a gradient-enhanced nonlocal part. However, in the latter approach an additional gradient parameter occurs to control the degree of regularization, which provides limited physical interpretation. Alternatively, the aforementioned numerical drawbacks may be overcome by applying viscous damage models, see e.g. Peña (2011, 2014). However, in large elastic vessels, where balloon angioplasty is typically applied, viscous effects play a rather minor role (see e.g. Fung, 1993). In the present contribution, a different approach to avoid possible mesh-dependency is followed based on a relaxed incremental variational damage formulation. Thereby, damage can be microscopically interpreted as a homogenization of a weakly and a strongly damaged phase and a convexified incremental stress potential can be constructed, see e.g. Francfort and Marigo (1993). In Francfort and Marigo (1998) also an application to fracture is provided. In Gürses and Miehe (2011) relaxation techniques are applied to continuum damage mechanics formulations by accounting for the generalized variational approach by Miehe (2002). Thereby, mesh-independency of finite element solutions could be shown when using relaxed potentials. However, the aforementioned models were derived in a small strain framework. As a first relaxed incremental variational damage formulation for the finite strain setup Balzani and Ortiz (2012) propose an approach which also shows mesh-independent

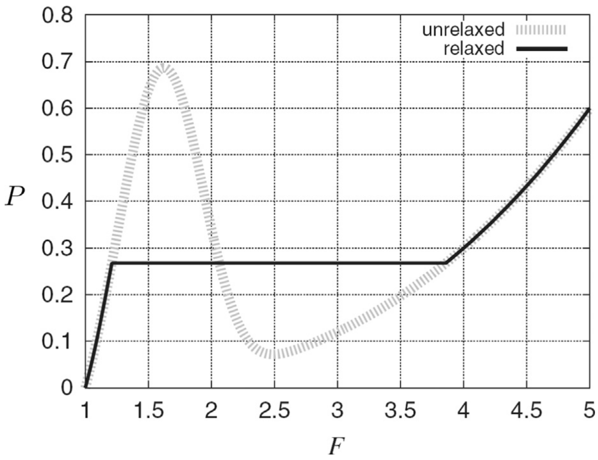

Fig. 1 – Exemplary stress–stretch response of an unrelaxed (continuum) damage formulation and of the associated relaxed model. Hereby, P and F denote the one-dimensional counterparts of the nominal stress and the deformation gradient, respectively. The relaxed formulation yields a constant stress response in the convexified regime.